

**Practice Exam 1**

1. Find the domain of  $\frac{x}{\sqrt{x^2 - x}}$ . Express your answer in interval notation.

2. Is the function  $f(x)$  one-to-one?

$$f(x) = \begin{cases} x^3 + 1 & \text{if } x < 0 \\ 2x & \text{if } x \geq 0 \end{cases}$$

3. Write the domain and range of  $f(x) = |x + 2| - 1$  in interval notation.

4. What is the average rate of change of  $f(x) = \log_2(x + 3)$  on the interval  $[1, 5]$ .

5. Compute the limits.

$$(i) \lim_{x \rightarrow 0^+} \frac{x}{|x|} =$$

$$(ii) \lim_{x \rightarrow 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x} =$$

$$(iii) \lim_{x \rightarrow \infty} \frac{2x^3}{4x^3 + x^2} =$$

$$(iv) \lim_{x \rightarrow \infty} e^{-x} =$$

$$(v) \lim_{x \rightarrow -2} \frac{x}{(x+2)^2} =$$

6. Let  $f(x) = 2x + b$ , where  $b$  is a constant, and note  $\lim_{x \rightarrow 2} f(x) = 4 + b$ . Find the largest  $\delta > 0$  such that, for  $\varepsilon = 2$ ,

$$|x - 2| < \delta \quad \implies \quad |f(x) - (4 + b)| < \varepsilon.$$

7. For what values of  $a$  is  $f(x)$  continuous for all real  $x$ .

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ 2x + a & \text{if } x \geq 1 \end{cases}$$

8. Find derivative of  $f(x) = \frac{1}{\sqrt{x}}$  at  $x = 1$  using the definition of the derivative.

9. Suppose  $f(1) = 2$  and  $f'(1) = 3$ , for a function  $f(x)$  which is differentiable at  $x = 1$ . Find the equation of the line tangent to the graph  $y = f(x)$  at  $x = 1$ .