Math 1501	Calculus I	Fall '13

Practice Exam 2

1. Find $\frac{dy}{dx}$ if $y = \cos^x(x)$.

2. Use the properties of natural log to simplify first, then find y' where $y = \ln\left(\dfrac{2x^{3/2}}{x^2+1}\right)$.

3. A triangle with hypotenuse length 5, height 4 and base x is shrinking as $\frac{dx}{dt} = -2$. How fast is the area shrinking when $x = 3$?

4. Write the sum in sigma notation.

$$1 - \frac{3}{2} + \frac{9}{4} - \frac{27}{8} + \frac{81}{10} - \cdots$$

5. Evaluate. $\displaystyle\sum_{i=1}^{5} \frac{2i}{(\sqrt{3})^i}$

6. A snowball in the shape of a sphere of radius r is melting such that $\frac{dr}{dt} = -1.5$ cm/hr. How fast is the volume changing when $r = 3$ cm?

7. Find the linearization of $\sin(x)$ at $x = 0$ and use it to approximate $\sin(0.2)$.

8. Let $f(x) = \sqrt{x}$, and note that the slope of the secant line passing through $(1, 1)$ and $(9, 3)$ is $m = 1/4$. The mean value theorem asserts that there exists a c in the interval $[1, 9]$ such that $f'(c) = 1/4$. Since the function f is increasing, this c is unique. Find it.

9. Find and classify the critical points of $f(x) = x^3(x^2 + 1)^2$. What is the absolute maximum and absolute minimum of $f(x)$ on the interval $[-1, 4]$?

10. What is the indefinite integral of $x^2 - 3x + \pi - \frac{1}{x}$?

11. Find a Riemann sum which approximates the area under the curve $y = \sin(x)$ between $x = 0$ and $x = 2\pi$ using $n = 6$ rectangles using left-endpoint approximation and evaluate.

12. Suppose $f'(x) = x^2(x-1)(x+2)$. Find the critical points of f. Find the intervals where f is increasing and where f is decreasing, and the intervals where f is concave up and concave down. Use the second derivative test to classify the critical points of f as local maxima/minima or neither.