| Math 1501 | Calculus I | Fall '13 |
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# Practice Exam 2

**1.** Find $\frac{dy}{dx}$ if $y = \cos^x(x)$.

$$\ln y = \ln(\cos^x(x)) = x \ln(\cos(x))$$

$$\frac{1}{y} y' = \ln(\cos(x)) + \frac{x}{\cos(x)} * -\sin(x)$$

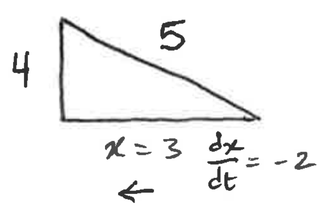
$$y' = \boxed{\cos^x(x)\left(\ln(\cos(x)) - x\tan(x)\right)}$$

**2.** Use the properties of natural log to simplify first, then find $y'$ where $y = \ln\left(\frac{2x^{3/2}}{x^2+1}\right)$.

$$y = \ln(2) + \frac{3}{2}\ln(x) - \ln(x^2+1)$$

$$y' = 0 + \frac{3}{2}\cdot\frac{1}{x} - \frac{2x}{x^2+1} = \boxed{\frac{3}{2x} - \frac{2x}{x^2+1}}$$

**3.** A right triangle with [redacted], height 4 and base $x$ is shrinking as $\frac{dx}{dt} = -2$. How fast is the area shrinking when $x = 3$?




$$A = \frac{1}{2}\cdot 4 \cdot x = 2x$$

$$\frac{dA}{dt} = 2\cdot\frac{dx}{dt} = 2(-2) = \boxed{-4}$$

**4.** Write the sum in sigma notation.

$$1 - \frac{3/2\;9}{+\;4} - \frac{27}{8} + \frac{81}{16} - \cdots$$

$$\boxed{\sum_{k=0}^{\infty} (-1)^k \left(\frac{3}{2}\right)^k}$$

**5.** Evaluate. $\displaystyle\sum_{i=1}^{5} \frac{2i}{(\sqrt{3})^i}$ $= \dfrac{2}{\sqrt{3}} + \dfrac{4}{\sqrt{3}^2} + \dfrac{6}{\sqrt{3}^3} + \dfrac{8}{\sqrt{3}^4} + \dfrac{10}{\sqrt{3}^5}$

$$= \frac{2}{\sqrt{3}} + \frac{4}{3} + \frac{6}{3\sqrt{3}} + \frac{8}{3^2} + \frac{10}{3^2\sqrt{3}}$$

$$= \frac{18 + 12\sqrt{3} + 18 + 8\sqrt{3} + 10}{9\sqrt{3}} = \boxed{\frac{46 + 20\sqrt{3}}{9\sqrt{3}}}$$

**6.** A snowball in the shape of a sphere of radius $r$ is melting such that $\frac{dr}{dt} = -1.5$ cm/hr. How fast is the volume changing when $r = 3$ cm?


$\dfrac{dr}{dt} = -1.5$

$\dfrac{dV}{dt} = ?$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(3)^2 \cdot (-1.5)$$

7. Find the linearization of $\sin(x)$ at $x = 0$ and use it to approximate $\sin(0.2)$.

$L(x) = f(a) + f'(a)(x-a)$ $\quad f(x) = \sin(x),\ a = 0$

$L(x) = \sin(0) + \cos(0)(x-0)$

$L(x) = 0 + 1(x-0)$

$L(x) = x$

$L(0.2) = 0.2$ $\quad$ So $\quad \sin(0.2) \approx L(0.2) = \boxed{0.2}$

8. Let $f(x) = \sqrt{x}$, and note that the slope of the secant line passing through $(1,1)$ and $(9,3)$ is $m = 1/4$. The mean value theorem asserts that there exists a $c$ in the interval $[1,9]$ such that $f'(c) = 1/4$. Since the function $f$ is increasing, this $c$ is unique. Find it.

$f'(x) = \frac{1}{2\sqrt{x}} = 1/4 \Rightarrow 1 = \frac{1}{2}\sqrt{x} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

$\boxed{c = 4}$

9. Find and classify the critical points of $f(x) = x^3(x^2+1)^2$. What is the absolute maximum and absolute minimum of $f(x)$ on the interval $[-1, 4]$?

$f'(x) = 3x^2(x^2+1)^2 + x^3 * 2(x^2+1) * 2x = 0$

$\Rightarrow (x^2+1)(3x^2(x^2+1) + 4x^4) = 0$

$\Rightarrow (x^2+1)(3x^4 + 3x^2 + 4x^4) = 0$

$\Rightarrow x^2(x^2+1)(7x^2+3) = 0$ (impossible). unless $x = 0$

No critical points, except at $x = 0$.

$f(-1) = -4$

$f(0) = 0$

$f(4) = 4^3 \cdot 17^2$

ABS min at $-4$ @ $x = -1$

ABS max of $4^3 \cdot 17^2$ @ $x = 4$.

**10.** What is the indefinite integral of $x^2 - 3x + \pi - \frac{1}{x}$?

$$\int x^2 - 3x + \pi - \frac{1}{x} \, dx$$

$$= \boxed{\frac{1}{3}x^3 - \frac{3}{2}x^2 + \pi x - \ln|x| + C}$$

**11.** Find a Riemann sum which approximates the area under the curve $y = \sin(x)$ between $x = 0$ and $x = 2\pi$ using $n = 6$ rectangles using left-endpoint approximation and evaluate.

SKIP

**12.** Suppose $f'(x) = x^2(x-1)(x+2)$. Find the critical points of $f$. Find the intervals where $f$ is increasing and where $f$ is decreasing, and the intervals where $f$ is concave up and concave down. Use the second derivative test to classify the critical points of $f$ as local maxima/minima or neither.

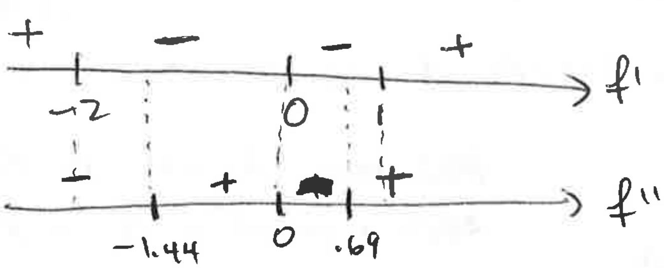
$f'(x) = x^2(x-1)(x+2) = 0$

$f'(x) = x^2(x^2+x-2)$
$= x^4 + x^3 - 2x^2$

$\Rightarrow x = 0, 1, -2.$ crit points.

$f''(x) = 4x^3 - 3x^2 - 4x = x(4x^2 + 3x - 4) = 0$

$\Rightarrow x = 0, \quad x = \frac{-3 \pm \sqrt{73}}{8} = -1.44, .69$




inc on $(-\infty, -2) \cup (1, \infty)$
dec on $(-2, 0) \cup (0, 1)$
CCU on $(-1.44, 0) \cup (.69, \infty)$
CCD on $(-\infty, -1.44) \cup (0, .69)$
$x = -2$ Max, $x = 1$ MIN
$x = 0$ inflection pt