Math 1501 **Exam 2** Fall 2013

Instructor: Sal Barone

Name: KEY

GT username: ____________________

Circle your TA/section: (N1) Daniel (N2) Rebecca (C1) Rachel (C2) Lily

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work to receive full credit.
4. Write your answers in the box provided.
5. Good luck!

Page	Max. Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. Find $f''(x)$ if $f(x) = \ln\left(\dfrac{3xe^{x^2}}{\sec(3x)}\right)$. (10 pts.)

$$f(x) = \ln 3 + \ln x + x^2 - \ln(\sec 3x)$$

$$f'(x) = \frac{1}{x} + 2x - \frac{3\,\cancel{\sec 3x}\tan 3x}{\cancel{\sec 3x}} = \frac{1}{x} + 2x - 3\tan 3x$$

$$f''(x) = \frac{-1}{x^2} + 2 - 9\sec^2 3x$$

$$\boxed{\frac{-1}{x^2} + 2 - 9\sec^2 3x}$$

2. Suppose f is continuous on $(-\infty, \infty)$ and differentiable for all x except at $x = 0$ and $x = 1$, and that

$$f'(x) = \begin{cases} 3e^{-x} & \text{if } x < 0, \\ 0 & \text{if } 0 < x < 1, \\ 3 & \text{if } x > 1. \end{cases}$$

Compute the average rate of change of f on the interval $[-1, 2]$. (10 pts.)

One approach:

$$f(x) = \begin{cases} -3e^{-x} + C_1 & x < 0 \\ C_2 & 0 \le x \le 1 \\ 3x + C_3 & x > 1 \end{cases}$$

Find C_1, C_2, C_3 which make f continuous.

e.g. $C_1 = 0$
$C_2 = -3$
$C_3 = -6$

So this f works:

$$f(x) = \begin{cases} -3e^{-x} & x < 0 \\ -3 & 0 \le x \le 1 \\ 3x - 6 & x > 1 \end{cases}$$

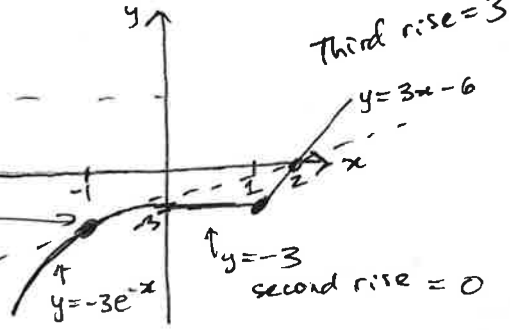
$$\boxed{e}$$

$$\frac{f(2) - f(-1)}{2 - (-1)} = \frac{0 - (-3e^{-(-1)})}{3} = \frac{3e}{3} = e$$

2nd approach could be:

$$\frac{\text{total rise}}{\text{total run}} = \frac{-3 + 3e + 3}{3} = e$$

first rise
$-3 - (-3e)$
$= -3 + 3e$



Third rise = 3

second rise = 0

3. Let $f(x) = \dfrac{3e^{x^2}}{2x-1}$.

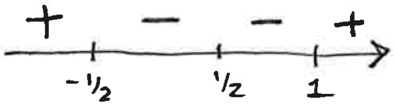
(i) Find and classify the critical points of $f(x)$ as a local maximum, local minimum, or neither. (14 pts.)

$$f'(x) = \frac{(2x-1)(6xe^{x^2}) - 3e^{x^2}(2)}{(2x-1)^2} = 0 \Rightarrow 6e^{x^2}(2x^2-x-1)=0$$

$$\Rightarrow 2x^2-x-1=0$$

$$\Rightarrow (2x+1)(x-1)=0$$

$$\Rightarrow x = -1/2,\ 1$$



Critical point @ $x=1$.

$f'<0$ when $\frac{1}{2}<x<1$

$f'>0$ when $x>1$

Critical point @ $x=-1/2$

$f'>0$ when $x<-1/2$

$f'<0$ when $-1/2<x<0$

Crit point at $x=-1/2$ is a MAX

Crit point at $x=1$ is a MIN

(ii) Find the absolute maximum and absolute minimum values of the function $f(x)$ on the interval $[-1, 0]$. (6 pts.)

on $[-1,0]$.

$$f(-1) = \frac{3e}{-3} = -e$$

$$-3 < -e < -\tfrac{3}{2}\sqrt[4]{e}$$

$$f(0) = \frac{3}{-1} = -3$$

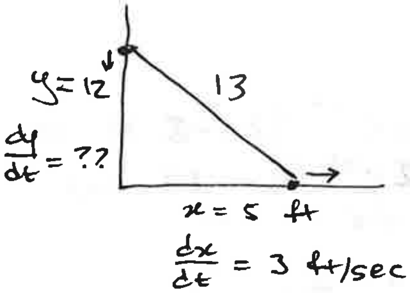
$$f(-1/2) = \frac{3e^{1/4}}{-2} = -\tfrac{3}{2}\sqrt[4]{e}$$

ABS MIN of -3 @ $x=0$

ABS MAX of $\frac{-3}{2}\sqrt[4]{e}$ @ $x=-1/2$

4. A 13-ft ladder is leaning against the wall as the base starts to slide away. By the time the base is 5 ft from the wall, the base is moving at the rate of 3 ft/sec.

(i) How fast is the side of the ladder sliding down the wall at this time? (10 pts.)



$$x^2 + y^2 = 169$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x}{y} \cdot \frac{dx}{dt} = \frac{-5}{12}(3) = -5/4$$

$$\boxed{-1.25 \text{ ft/sec}}$$

(ii) At what rate is the area of the triangle formed by the ladder, ground, and wall changing at this time? (10 pts.)

$$A = \frac{1}{2} x y$$

$$\frac{dA}{dt} = \frac{1}{2}\left(x \cdot \frac{dy}{dt} + \frac{dx}{dt} \cdot y\right)$$

$$= \frac{1}{2}[5(-1.25) + (3)(12)]$$

$$= \frac{1}{2}[-6.25 + 36]$$

$$= \frac{1}{2}[29.75]$$

$$= 14.875$$

$$\boxed{14.875 \text{ ft}^2/\text{sec}}$$

5. Suppose f is defined on $[0, \pi]$ and $f'(x) = \sin(x)\cos(x)$. Then $y = f(x)$ is concave up on the interval(s) (10 pts.)

$f''(x) = \sin(x) \ast -\sin(x) + \cos(x)\cos(x)$

$f''(x) = -\sin^2(x) + \cos^2(x) = 0$

$\Rightarrow \cos^2(x) = \sin^2(x)$

$\Rightarrow 1 = \tan^2(x)$

$\Rightarrow \tan(x) = \pm 1.$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$ (in domain)

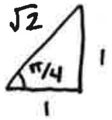
(A) $\left(\frac{\pi}{2}, \pi\right)$

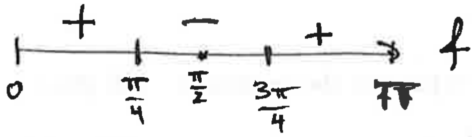
(B) $\left(0, \frac{\pi}{3}\right), \left(\frac{2\pi}{3}, \pi\right)$

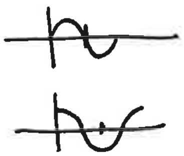
(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

[✓ **(D)** $\left(0, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \pi\right)$]

(E) None of the above.







6. Which of the following are always true for a function f from $\mathbb{R}$ to $\mathbb{R}$? (10 pts.)

✓**(I)** If f is a polynomial of odd degree, then there exists a root c of f such that $f(c) = 0$.

(II) If $f'(c) = 0$ and $\underline{f''(c) < 0}$, then $(c, f(c))$ is a local ~~minimum~~. maximum

✓**(III)** Either f is increasing on the interval (a, b) or f is decreasing on the interval (a, b) if f is differentiable and has no critical points in (a, b).

(A) I only.

(B) II only.

(C) I & II.

[✓ **(D)** I & III.]

(E) All are true.

7. Let $f(x) = x - \sqrt{x}$. Using the linearization of $f(x)$ at $x = 9$ one finds that $f(7)$ is approximately (10 pts.)

$$L(x) = f(a) + f'(a)(x-a)$$

$$f'(x) = 1 - \frac{1}{2\sqrt{x}}$$

$$f'(9) = 1 - \frac{1}{6} = 5/6$$

$$L(x) = (9-3) + \frac{5}{6}(x-9)$$

$$L(x) = 6 + \frac{5}{6}(x-9)$$

$$L(7) = 6 + \frac{5}{6}(7-9)$$

$$f(7) \approx L(7) = \frac{36}{6} - \frac{10}{6} = \frac{26}{6} = \frac{13}{3}$$

(A) $\frac{23}{3}$.

(B) $\frac{14}{3}$.

[✓ (C) $\frac{13}{3}$.]

(D) $\frac{25}{6}$.

(E) $\frac{23}{6}$.

8. Which of the following sums in sigma notation expresses the sequence (10 pts.)

$$\frac{1}{2} - \frac{3}{4} + \frac{5}{8} - \frac{7}{16} + \frac{9}{32} - \cdots$$

$$\sum_{k=1}^{\infty} \frac{2k-1}{2^k} * (-1)^{k+1}$$

(A) $\sum_{k=0}^{\infty}(\text{-}1)^k 2^{k+1}(2k+1)$

(B) $\sum_{k=0}^{\infty}(\text{-}1)^{k+1} 2^{-k}(2k-1)$

(C) $\sum_{k=1}^{\infty}(\text{-}1)^k 2^{-k+1}(2k+1)$

(D) $\sum_{k=0}^{\infty}(\text{-}1)^k 2^{-k+1}(2k+1)$

[✓ (E) $\sum_{k=1}^{\infty}(\text{-}1)^{k+1} 2^{-k}(2k-1)$]