Practice Exam 3

1. Show that $f(x) = \frac{3}{4x^2+2}$ satisfies $f(x) = O(1)$.

2. Find the particular solution to the separable differential equation $y' = \sqrt{\frac{x}{y}}$ satisfying $f(0) = \sqrt{3}$.

3. Let R be the region bounded by the curves $y = x^2(x - 1)$, $y = 4$ and $x = 0$.

(i) Find the area of R.

(ii) Find the x-value of the center of mass of R.

(iii) Find the volume of the solid obtained by rotating R about the y-axis.

4. Consider the curve $y = x^{3/2}$ between $x = 0$ to $x = 2$. Find the arc length of the curve.

5. Find the surface area of the surface obtained by rotating the curve $y = x^3$, $0 \leq x \leq 2$, about the x-axis.

6. Show that $y = \int_1^x \frac{1}{t}\, dt \ - \ 2$ satisfies the initial value problem $\frac{dy}{dx} = \frac{1}{x}$ and $y(1) = -2$.

7. Consider the sequences $a_n = \frac{n^2+1}{n+1}$ and $b_n = \ln(n) - \ln(n+1)$. Do the sequences a_n and b_n converge or diverge?

8. Suppose the force to move an object over 5 feet is given by $F(x) = \frac{x^2}{\sqrt{30-x^2}}$, $0 \leq x \leq 5$, where x is the distance from start. Find the work needed to move the object 5 feet.

9. Integrate $\displaystyle\int \tan^4 x \, dx$.