Final problem set #1

1. Find the domain, intervals where the function is continuous, and intervals where the function is differentiable for the following functions:

$f(x) = x^2$	$g(x) = \sqrt{x}$	$h(x) = \ln(x)$
$f \circ g$	$h(x-5)+2$	$f(\tan x)$
$g \circ f$	f/h	$g(\tan x)$
$h \circ h$	$h(\frac{x}{1-x})$	$h(\tan x)$

2. Differentiate the 12 functions from problem 1.

3. Find the limits of the following sequences as $n \to \infty$.

$$\left(3 + \frac{2}{n}\right)^n \qquad \sin\left(\frac{\pi n}{n+1}\right) \qquad \ln\left(\frac{n^2}{e^n}\right) \qquad \ln\left(\frac{n^2+2n-1}{3n^2-n+1}\right)$$

4. Find the general anti-derivative of the following functions:

$\dfrac{\ln(x^2)}{x}$	$2x\ln(x)$	$\dfrac{2x+5}{x^2+5x+6}$
$\dfrac{\ln(x^2)}{x^2}$	$\sin^5(x)\cos^2(x)$	$\dfrac{x+1}{x^2+5x+6}$

5. Find $\frac{dy}{dx}$ (+simplify) if

$$y^2 + \ln(xy^2) - 4x + 2 = 0.$$

6. Find the particular solution $y = f(x)$ to the differential equation $y'e^{y+x^2} = -2x$ satisfying $y(0) = \ln 2$.

7. Find and classify the critical points of $f(x)$ in $[0, 2\pi]$ as local maximum, minimum, or neither.

$$f(x) = \int_0^x \cos(2t)\, dt.$$

8. What is the volume of a bucket which is 8 dm (decimeter) in height and has a parabolic cross-section through the middle given by $y = x^2$?

9. If the metal that the bucket is made of has density $\dfrac{30}{\pi(17^{3/2}-1)}$ lbs/dm^2, then how much does the bucket weigh?

10. Suppose now the bucket is filled with water (recall that water weighs 2.2 lbs per cubic decimeter) and the bucket has a steady drip, which leaks half of what is left in the bucket every minute. The bucket is being lifted up to a tall window 80 feet in the air. Find the work done to lift the leaky bucket if the bucket is being lifted 20 feet per minute.