Final problem set #2

1. Let $f(x) = \ln(\frac{3xe^{x^3}}{1-2x})$. Find the domain of f.

2. Find $f(1), f(-1), f'(1), f'(-1)$, if they exist, for the function f from problem 1. Also, find the general anti-derivative $\int f(x)\,dx$.

3. Let $g(x) = \frac{xe^x - 3e^x}{9x - 3x^2}$. Find $\lim_{x\to\infty} g(x)$, $\lim_{x\to-\infty} g(x)$ and $\lim_{x\to 3^-} g(x)$.

4. Find the equation of the tangent line to the circle $x^2 + y^2 = 25$ passing through the point $(3, 4)$.

5. When a circular plate is heated in an oven, its radius increases at a rate of 0.02 cm/min. How fast is the area of the plate increasing when the radius is 50 cm?

6. A drop of mist is (essentially) a perfect sphere which collects additional moisture, through condensation, at a rate proportional to its surface area. Show that in this case the drop's radius is increasing at a constant rate.

7. Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The ratio of the height of the pile to the base diameter is always 3/8. How fast are the height and radius changing when the pile is 4 m high?

8. Find the linearization of $y = x\sqrt{1-x^2}$ at $x = 1/2$. What is dy? *(Recall: $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ and $\Delta y \approx f'(x_0)\Delta x$ near x_0, and $dy = f'(x)\,dx$)*

9. A surveyor, standing 30 ft from the base of a building, measures the angle of elevation to the top of the building to be $75\,°$. How accurately must the angle be measured to ensure that the error in estimating the height is less than 4%?

10. A certain particle has position $s = \sin(t + \sqrt{t+1})$ at time t. Find the position, velocity, speed, and acceleration of the particle at time $t = 1$.

11. Find the absolute max and absolute min (if they exist) of the functions $y = |x|$ and $y = \frac{6}{x^2+1}$ on the interval $(-1, 2)$.

12. Suppose $f'(2) = 3$ and $f'(x) = 0$ for all x. Find f.

13. For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?