

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Linearization

$$L(x) = f(a) + f'(a)(x - a)$$

Derivatives

$$\frac{dy}{dx} \sin(x) = \cos(x)$$

$$\frac{dy}{dx} \cos(x) = -\sin(x)$$

$$\frac{dy}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{dy}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{dy}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{dy}{dx} \tan(x) = \sec^2(x)$$

$$\frac{dy}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} \operatorname{arcsec}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{dy}{dx} \arctan(x) = \frac{1}{1+x^2}$$

Product Rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Integrals

$$\int b^{ax} dx = \frac{1}{a \ln(b)} b^{ax} + C$$

$$\int \ln(x) dx = x \ln(x) - x + C$$

$$\int \tan(x) dx = -\ln|\cos(x)| + C$$

$$\int \csc(x) dx = -\ln|\csc(x) + \cot(x)| + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int u dv = uv - \int v du$$

Area

$$A = \int_a^b (\text{top}) - (\text{bottom}) dx$$

Trigonometric Substitutions

$$\sqrt{a^2 + x^2} \quad x = a \tan \theta$$

$$\sqrt{a^2 - x^2} \quad x = a \sin \theta$$

$$\sqrt{x^2 - a^2} \quad x = a \sec \theta$$

Volume by Revolution

Disk Method:

$$V = \int_a^b \pi [R(x)]^2 dx = \int_a^b \pi ([R(x)]^2 - [r(x)]^2) dx$$

Shell Method:

$$V = \int_a^b 2\pi(\text{shell radius})(\text{shell height}) dx$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

Surface Area

$$SA = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

Exponential Growth

$$A = A_0 e^{kt}$$

Work

$$W = Fd = \int_a^b F(x) dx$$

Hooke's Law

$$F = kx$$

Center of Mass

$$M_y = \int \tilde{x} dm = \int_a^b x \delta [f(x) - g(x)] dx$$

$$\begin{aligned} M_x &= \int \tilde{y} dm = \int_a^b \frac{1}{2} [f(x) + g(x)] \times \delta [f(x) - g(x)] dx \\ &= \int_a^b \frac{\delta}{2} [f^2(x) - g^2(x)] dx \end{aligned}$$

$$M = \int dm = \int_a^b \delta [f(x) - g(x)] dx$$

$$\bar{x} = \frac{M_y}{M}$$

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