Math 1501 Calc I

Fall 2013

Lesson 1 - Lesson 8

Instructor: Sal Barone

School of Mathematics
Georgia Tech

August 19 - August 6, 2013
(updated September 1, 2013)

First day

- Syllabus, homework set, practice quizzes & exams
  - http://people.math.gatech.edu/~sbarone7/ma1501.html

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- MyLab online homework
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- MyLab online homework
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- Important dates:
  - First homework due Tuesday, August 20 (tomorrow)
  - First quiz Thursday, August 22
  - Exam 1 Tuesday, September 10

L1: FUNCTIONS, DOMAIN & RANGE

Covered sections: §1.1

Quiz 1 (L1-L2) Thursday, August 22

L1: Functions, domain & range

Definition

A *function* f from $\mathbb{R}$ to $\mathbb{R}$ is a rule that assigns to each number x a unique number $f(x)$.

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The *domain* D of a function f is the subset of $\mathbb{R}$ where the function is defined. The *range* R of f is the subset of $\mathbb{R}$ consisting of all the function values which are hit by f, that is, those y values such that $y = f(x)$ for some x in the domain.

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Definition
A function is *even* if $f(x) = f(-x)$ for all x in the domain of f.
A function f is *odd* if $f(x) = -f(x)$ for all x in its domain.

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add x and y	$f(x, y) = x + y$
add 2 to x	$f(x) = 2 + x$
find the point on the unit circle that makes an angle of θ with the x-axis	$f(\theta) = (\cos\theta, \sin\theta)$
find the y-value of the point on the unit circle that makes an angle of θ with the x-axis	$f(\theta) = \sin\theta$

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round up x to the nearest integer

$$f(x) = \lceil x \rceil$$

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Exponential	$y = c \cdot a^{kx}$	$y = 2e^{3x}$
Logarithmic	$y = c \cdot \log_b ax$	$y = \log_2 x$

L2: COMBINING FUNCTIONS, TRIG FUNCTIONS

Covered sections: §1.2 & §1.3

Quiz 1 Thursday (L1-L2), August 22

L2: Combining functions, trig functions

Definition

The *composition* of two functions, denoted by $f \circ g$ and read as "f circle g" is the function defined by $f \circ g := f(g(x))$.

L2: Combining functions, trig functions

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For $c > 1$ the graph $y = f(x)$ is scaled as follows

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For $c = -1$ the graph is reflected

across x-axis:	$y = -f(x)$	$y = -x^2$

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across y-axis:	$y = f(-x)$	$y = (-x)^2$

L2: Combining functions, trig functions

Trigonometric functions

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \cos\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \qquad \tan\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

L2: COMBINING FUNCTIONS, TRIG FUNCTIONS

Trigonometric identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
- $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

L3: Exponential, inverse, and logarithmic functions

Covered sections: §1.5 & §1.6

Quiz 2 Thursday, August 29

L3: Exponential, inverse, and logarithmic functions

Exponent rules

- $a^x \cdot a^y = a^{x+y}$
- $(a^x)^y = a^{xy}$
- $a^x \cdot b^x = (ab)^x$
- $a^{-x} = \frac{1}{a^x}$

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- $a^{-x} = \frac{1}{a^x}$
- $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$
- $\frac{a^x}{a^y} = a^{x-y}$
- $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

L3: Exponential, inverse, and logarithmic functions

Inverse functions

Definition

f is *one-to-one* if every value y in the range of f is hit exactly once, in symbols

$$f(x_1) = f(x_2) \implies x_1 = x_2.$$

L3: Exponential, inverse, and logarithmic functions

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Definition
If f is one-to-one then f has an *inverse function*, denoted f^{-1}, which is the function which "undoes" whatever f does. In symbols $f \circ f^{-1}(x) = x$ and $f^{-1} \circ f(x) = x$.

L4: Average rate of change and Limits

Covered sections: §2.1 & §2.2

Quiz 2 Thursday, August 29

L4: Average rate of change and Limits

Definition

The *average rate of change* of the function $f(x)$ over the interval $[a, b]$ is given by the equation

$$\frac{f(b) - f(a)}{b - a}.$$

L4: Average rate of change and Limits

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Definition

The *line tangent to the curve $y = f(x)$ at $x = a$* is the line which touches the curve $y = f(x)$ at $x = a$ and such that the line has the same direction as $y = f(x)$ at $x = a$.

L4: Average rate of change and Limits

What is the **slope** of the line tangent to the curve $y = x^2$ at $x = 1$? How can you approximate it?

length of interval h	average rate of change

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1	$\frac{f(2)-f(1)}{2-1} = \frac{4-1}{1} = 3$
.5	$\frac{f(1.5)-f(1)}{1.5-1} = \frac{2.25-1}{.5} = 2.5$

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.25	$\frac{f(1.25)-f(1)}{1.25-1} = \frac{1.5625}{.25} = 2.25$
.1	$\frac{f(1.1)-f(1)}{1.1-1} = \frac{1.21-1}{.1} = 2.1$

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Definition
The *limit* of a function $y = f(x)$ as x approaches a is y_0 if the following happens:
$f(x)$ gets closer and closer to y_0 as x gets closer and closer to a.

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In this case write

$$\lim_{x \to a} f(x) = y_0$$

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In this case write

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or equivalently

$$f(x) \underset{x \to a}{\longrightarrow} y_0.$$

L4: Average rate of change and Limits

An example: what is the limit?

$$\lim_{x \to -1} \frac{x^2 - 6x + 9}{x^2 - 9}$$

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x-value	$f(x)$-value		
3.1	$\frac{(3.1)^2 - 6(3.1) + 9}{(3.1)^2 - 9}$	$=$	$.0163\ldots$

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3.1	$\frac{(3.1)^2 - 6(3.1) + 9}{(3.1)^2 - 9}$	$=$	$.0163\ldots$
2.9	$\frac{(2.9)^2 - 6(2.9) + 9}{(2.9)^2 - 9}$	$=$	$-.0163\ldots$

L4: Average rate of change and Limits

An example: what is the limit?

$$\lim_{x \to \text{-}1} \frac{x^2 - 6x + 9}{x^2 - 9}$$

x-value	$f(x)$-value		
3.1	$\frac{(3.1)^2-6(3.1)+9}{(3.1)^2-9}$	$=$	$.0163\ldots$
2.9	$\frac{(2.9)^2-6(2.9)+9}{(2.9)^2-9}$	$=$	$-.0163\ldots$
3.01	$\frac{(3.01)^2-6(3.01)+9}{(3.01)^2-9}$	$=$	$.0016\ldots$

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3.01	$\frac{(3.01)^2 - 6(3.01) + 9}{(3.01)^2 - 9}$	$=$	$.0016\ldots$
2.99	$\frac{(2.99)^2 - 6(2.99) + 9}{(2.99)^2 - 9}$	$=$	$-.0016\ldots$
3	$\frac{(3)^2 - 6(3) + 9}{(3)^2 - 9}$	$=$	$\frac{0}{0}$!

Limit rules:

- If $P(x) = a_n x^n + \ldots + a_1 x + a_0$ is a polynomial, then

$$\lim_{x \to a} P(x) = P(a).$$

L4: Average rate of change and Limits

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Example: $\lim_{x \to \text{-}2} x^2 - 3x + 1 = (\text{-}2)^2 - 3(\text{-}2) + 1 = 11.$

L4: Average rate of change and Limits

Limit rules:

- If $P(x) = a_n x^n + \ldots + a_1 x + a_0$ is a polynomial, then

$$\lim_{x \to a} P(x) = P(a).$$

- If $f(x) = \frac{P(x)}{Q(x)}$ is rational, so $P(x), Q(x)$ are polynomials, and $Q(a) \neq 0$, then

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}.$$

L4: Average rate of change and Limits

Limit rules:

- If $P(x) = a_n x^n + \ldots + a_1 x + a_0$ is a polynomial, then

$$\lim_{x \to a} P(x) = P(a).$$

- If $f(x) = \frac{P(x)}{Q(x)}$ is rational, so $P(x), Q(x)$ are polynomials, and $Q(a) \neq 0$, then

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}.$$

Example: $\lim_{x \to 1} \frac{x^2 - 1}{x + 3} = 0.$

More limit rules: If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

- $\lim_{x\to a} (f(x) + g(x)) = L + M$,
- $\lim_{x\to a} cf(x) = cL$,
- $\lim_{x\to a} (f(x) \cdot g(x)) = LM$,
- $\lim_{x\to a} \frac{f(x)}{g(x)} = L/M$, and
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L5: Precise definition of a limit[1]

Covered sections: §2.3

Quiz 2 Thursday, August 29

[1] no lecture slides

L6: LIMITS INVOLVING INFINITY, ONE SIDED LIMITS AND CONTINUITY[2]

Covered sections: §2.4, §2.5 & §2.6

Quiz 3 Thursday, September 5

[2] no lecture slides

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  - Why did I get this grade on a quiz?
  - The way I did this HW problem is different than the way you did it in class, is that ok?

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- TAs and I can give some pointers on good ways to study.

L7: Asymptotes and the definition of the derivative

Covered sections: §2.6 & §3.1

Quiz 3 Thursday, September 5

L8: The derivative as a function, equation of the tangent line

Covered sections: §3.2

Exam 1 Tuesday, September 10