Math 1501 Calc I
Fall 2013
Lesson 9 - Lesson 20

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School of Mathematics
Georgia Tech

August 19 - August 6, 2013
(updated October 4, 2013)

L9: Differentiation rules

Covered sections: §3.3 & §3.5

Exam 1 (L1-L8) Tuesday, September 10 (tomorrow)

Quiz 4 (L9-L10) Thursday, September 12

L9: Differentiation rules

Many symbols are used to notate the derivative of $y = f(x)$ with respect to x:

$$f' = y' = \frac{d}{dx}f = \frac{df}{dx} = \frac{dy}{dx}.$$

L9: DIFFERENTIATION RULES

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Remember the derivative $f'(x)$ is:

- the slope of the tangent line of $y = f(x)$ at x,
- the instantaneous rate of change of $f(x)$ at x,
- defined as the limit of the difference quotient of $f(x)$ near x.

L9: DIFFERENTIATION RULES

A *differentiation rule* is a rule that you can use to find f'.

Rule: (Power rule)

$\frac{d}{dx}(x^n) = nx^{n-1}$ *for any* n.

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Rule: (Constant multiple & Sum rules)

$$\frac{d}{dx}(cf) = c\frac{df}{dx} \qquad \frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx},$$

for any constant c *and functions* $f(x)$, $g(x)$.

L9: Differentiation rules

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for any functions $u(x)$ *and* $v(x)$.

Rule: (Quotient rule)

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

for any functions $u(x)$ *and* $v(x)$.

Definition

The derivative of the derivative of f is called the *second derivative* and denoted f''. The n-th derivative of f is the function obtained by taking the derivative of f n-times, and denoted $f^{(n)}$ if $n > 3$.

$$f, f', f'', f''', f^{(4)}, \ldots, f^{(n)}, \ldots$$

L9: DIFFERENTIATION RULES

The derivatives of trig functions:

$$(\sin\theta)' = \cos(\theta) \qquad (\csc\theta)' = -\csc(\theta)\cot(\theta)$$

$$(\cos\theta)' = -\sin(\theta) \qquad (\sec\theta)' = \sec(\theta)\tan(\theta)$$

$$(\tan\theta)' = \sec^2(\theta) \qquad (\cot\theta)' = -\csc^2(\theta)$$

L10: The chain rule & applications of the derivative

Covered sections: §3.4 & §3.6

Quiz 4 (L9-L10) Thursday, September 12
(tomorrow)

L10: THE CHAIN RULE & APPLICATIONS OF THE DERIVATIVE

Recall: the derivative f' of $f(x)$ is the instantaneous rate of change of f with respect to x.

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- If $s(t)$ is the position of a body, then s' is the body's velocity.

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- If $a(t)$ is the acceleration of a body, then a' is the body's jerk.

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- If $a(t)$ is the acceleration of a body, then a' is the body's jerk.

In Economics:

- If $c(x)$ is the cost to produce x units of a product, then c' is the marginal cost of production.

L10: The chain rule & applications of the derivative

Rule: (The chain rule)

The derivative of $g \circ f(x)$ is

$$f'(x) \cdot g'(f(x)),$$

or using alternate notation

$$\frac{df}{dx}(x) \cdot \frac{dg}{dx}(f(x)).$$

L11: Implicit differentiation

Covered sections: §3.7

Quiz 5 (L11-L13) Thursday, September 19

L11: Implicit differentiation

Exam 1 results are in:

AVG $= 84.4$ $\quad \sigma = 12.6 \quad N = 128$

Grade	Range	How many
A	90-100	51
B	80-90	39
C	70-80	21
D	60-70	11
F	0-50	6

Exam 1 page results breakdown:

Page	Avg %	σ
Page 1	71%	4.6
Page 2	87%	2.1
Page 3	91%	3.2
Page 4	89%	4.7
Page 5	84%	3.8

L12: DERIVATIVES OF INVERSES, LOGS, INVERSE TRIGONOMETRIC FUNCTIONS

Covered sections: §3.8 & §3.9

Quiz 5 (L11-L13) Thursday, September 19

L12: Derivatives of inverses, logs, inverse trigonometric functions

Rule: (Derivative of logarithms)

$$(\ln(x))' = \frac{1}{x} \qquad \textit{and} \qquad \left(\log_b(x)\right)' = \frac{1}{x\ln(b)}.$$

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Rule: (Derivative of inverse trig functions & inverse function-inverse Cofunction identitites)

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$$\frac{d}{du}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}$$

$$\frac{d}{du}\tan^{-1}(u) = \frac{1}{1+u^2}$$

$$\frac{d}{du}\sec^{-1}(u) = \frac{1}{|u|\sqrt{1-u^2}}$$

$$\cos^{-1}x = \pi/2 - \sin^{-1}x$$

$$\cot^{-1}x = \pi/2 - \tan^{-1}x$$

$$\cos^{-1}x = \pi/2 - \sec^{-1}x$$

L12: Derivatives of inverses, logs, inverse trigonometric functions

Rule: (Derivative of inverse trig functions & inverse function-inverse Cofunction identitites)

$$\frac{d}{du}\cos^{-1}(u) = \frac{-1}{\sqrt{1-u^2}}$$

$$\frac{d}{du}\cot^{-1}(u) = \frac{-1}{1+u^2}$$

$$\frac{d}{du}\csc^{-1}(u) = \frac{-1}{|u|\sqrt{1-u^2}}$$

$$\cos^{-1} x = \pi/2 - \sin^{-1} x$$

$$\cot^{-1} x = \pi/2 - \tan^{-1} x$$

$$\cos^{-1} x = \pi/2 - \sec^{-1} x$$

L12: Derivatives of inverses, logs, inverse trigonometric functions

Rule: (Derivative of inverses)

$$\left(f^{-1}\right)'(x) = \frac{1}{f'(f^{-1}(x))}$$

L13: RELATED RATES

Covered sections: §3.10

Quiz 5 (L11-L13) Thursday, September 19
(tomorrow)

L13: RELATED RATES

- The radius of a very large circle is decreasing at 3 in/sec. How is the area of the circle changing when the radius is 10 in?

L13: Related rates

- The radius of a very large circle is decreasing at 3 in/sec. How is the area of the circle changing when the radius is 10 in?
- Find the rate of change in area over time of a rectange of fixed radius 100 cm if the base is increasing at 2 cm/sec at the moment when the rectangle is a square.

L13: Related rates

General strategy for solving related rates problems:

- Draw and label a picture. Be sure to include variables.

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- What do you want? What do you have?
- Find an equation(s) which relates all important quantities.
- Take a derivative (usually $\frac{d}{dt}$) of both sides.
- Solve for what you 'want' from above.

L13: Related rates

- A rope running through a pulley at P and bearing a weight W is being pulled by a workers hand 5 ft above the ground. If the pulley is 25 ft above the ground, the rope is 45 ft long and the worker is walking rapidly away at a rate of 6 ft/sec, then how fast is the weight being lifted when the worker is 21 ft from under the weight?

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- You are watching NASCAR from a stand 132 ft from the track. As a car approaches moving 180 mi/hr (264 ft/sec), how fast will your head be turning when the car is right in front of you? What about a half second later after it has passed?

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- A girl flies a kite at a height of 300 ft while the wind is carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away?
- How fast is the angle the kite string makes with the ground changing at this time?

L14: LINEARIZATION AND DIFFERENTIALS & THE INTERMEDIATE VALUE THEOREM†, ROLLE'S THEOREM*, THE MEAN VALUE THEOREM*

Covered sections: §3.11 & §2.5†, §4.2*

Quiz 6 (L14-L16) Thursday, September 26

L14: Linearization and differentials & Rolle's Theorem*, The Mean Value Theorem*

Definition
The linearization of a differentiable function f at a is

$$L(x) = f(a) + f'(a)(x - a).$$

Fact: $L(x) \approx f(x)$ when $x \approx a$.

L14: Linearization and differentials & Rolle's Theorem*, The Mean Value Theorem*

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Example
Use linearization to approximate $\sqrt{1.5}$, $\sqrt{5}$, $\sqrt{15}$.

L14: Linearization and differentials & Rolle's Theorem*, The Mean Value Theorem*

Definition

Let $y = f(x)$ be a differentiable function. Then, the differential dx is an independent variable and the differential dy is

$$dy = f'(x)\,dx.$$

Fact: $\Delta y \approx dy$

L14: Linearization and differentials & Rolle's Theorem*, The Mean Value Theorem*

Actual change
$\Delta f = f(a + dx) - f(a)$

Estimated change
$df = f'(a)\, dx$

L14: LINEARIZATION AND DIFFERENTIALS & ROLLE'S THEOREM*, THE MEAN VALUE THEOREM*

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Example

Use differentials to approximate the increase in the area of a circle when the radius r increases from $a = 10$ to $a = 10.1$.

L14: Linearization and differentials & Rolle's Theorem*, The Mean Value Theorem*

Example

You want to calculate the depth of a well from the equation $s = 16t^2$ by timing how long it takes to hear a stone splash in the water below. How sensitive will your calculations be to a 0.1-second error in the measuring time if you hear the stone after 2 seconds? After 5 seconds?

Example

You want to calculate the depth of a well from the equation $s = 16t^2$ by timing how long it takes to hear a stone splash in the water below. How sensitive will your calculations be to a 0.1-second error in the measuring time if you hear the stone after 2 seconds? After 5 seconds?

At which time is your calculation more sensitive to error?

L14: LINEARIZATION AND DIFFERENTIALS & ROLLE'S THEOREM*, THE MEAN VALUE THEOREM*

Theorem (Intermediate Value Theorem)

If f is a continuous function on $[a, b]$ and y_0 is a value between $f(a)$ and $f(b)$, then there is some c in $[a, b]$ such that $f(c) = y_0$.

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Theorem (Rolle's Theorem)

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$ then there is at least one number c in (a, b) at which $f'(c) = 0$.

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Theorem (Mean Value Theorem)

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). Then there is at least one number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

L15: Absolute and local extreme values, critical points

Covered sections: §4.1

Quiz 6 (L14-L16) Thursday, September 26

L15: Absolute and local extreme values, critical points

- A function f has a *local maximum* value at c within its domain D if $f(x) \leq f(c)$ for all $x \in D$ in an open interval containing c.

L15: ABSOLUTE AND LOCAL EXTREME VALUES, CRITICAL POINTS

- A function f has a *local maximum* value at c within its domain D if $f(x) \leq f(c)$ for all $x \in D$ in an open interval containing c.
- A function f has a *local minimum* value at c within its domain D if $f(x) \geq f(c)$ for all $x \in D$ in an open interval containing c.

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Definition

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Local extrema of f only occur at critical points of f. Not all critical points correspond to local extrema.

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Definition
A point $x = a$ in the domain of f where $f'(a) = 0$ or f' is undefined is called a *critical point* of f.

Rule:
Local extrema of f only occur at critical points of f. Not all critical points correspond to local extrema.

Rule:
Absolute maxima of f occur at critical points of f or at the endpoints of a closed interval on which f is defined. Every continuous function f defined on a closed interval $[a, b]$ has an absolute maximum AND an abolute minimum value, which are obtained either at a, b, or a critical point c in (a, b).

L16: Intervals of increase, decrease and the First Derivative Test

Covered sections: §4.3

Quiz 6 (L14-L16) Thursday, September 26
(tomorrow)

L17: Concavity & Curve sketching, 2nd derivative test

Covered sections: §4.4

Quiz 7 (L17-L19) Thursday, October 3

L17: Concavity & Curve sketching, 2nd derivative test

Definition
If $f''(c) > 0$ then we say that f is *concave up* at $x = c$.

If $f''(c) < 0$ then we say that f is *concave down* at $x = c$.

If $x = c$ is a value where the concavity changes, goes from up to down or from down to up, then we say that $x = c$ is an *inflection point*.

L17: CONCAVITY & CURVE SKETCHING, 2ND DERIVATIVE TEST

Rule: (2nd derivative test)

If $x = c$ is a critical point of f and $f''(c) > 0$, then $x = c$ is a local minimum.

If $x = c$ is a critical point of f and $f''(c) < 0$, then $x = c$ is a local maximum.

L18: APPLIED OPTIMIZATION (+L'HÔPITALS RULE)

Covered sections: §4.6

Quiz 7 (L17-L19) Thursday, October 3

L18: Applied optimization (+L'Hôpitals Rule)

Rule: (L'Hôpitals rule)

If $f(a)/g(a)$ *is indeterminate of the form* $0/0$ *then*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

L18: APPLIED OPTIMIZATION (+L'HÔPITALS RULE)

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$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

Example

$\lim_{x\to 0}\frac{1-\cos x}{x^2}$

L18: Applied optimization

Now some optimization word problems:

Example

What are the dimensions of the smallest can which has a volume of 1024 cm^3?

L18: Applied optimization

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Example

A rectangle is to be inscribed in a circle of radius 3. Where should the vertices of the rectangle go to maximize the area of the rectangle?

L18: APPLIED OPTIMIZATION

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Example

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Example

A rectangle is to be inscribed in a circle of radius 3. Where should the vertices of the rectangle go to maximize the area of the rectangle?

Example

Suppose a company gets \$9 for each widget it sells, and the cost to make x widgets is $c(x) = x^3 - 6x^2 + 15x$. How many widgets should the company try to sell in order to maximize profit?

Example

You are designing a poster to contain 50 in^2 of printed material with 4-in margins at the top and bottom and 2-in margins on the sides. What are the smallest dimensions the paper could have?

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What dimensions of a box with square sides and a girth of 64 in (girth = length around) yield the maximum volume?

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Example

What dimensions of a box with square sides and a girth of 64 in (girth = length around) yield the maximum volume?

Example

An $8'$ wall stands $27'$ from a castle surrounded by a moat. What is the shortest ladder than can be put on the ground outside the wall and reach over the moat to the castle?

L19: ANTIDERIVATIVES

Covered sections: §4.8

Quiz 7 (L17-L19) Thursday, October 3
(tomorrow)

EXAM 2 (L9-L20) Thursday, October 10

L19: Antiderivatives

Definition

The *antiderivative* of $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$. The most general antiderivative of f is $F(x) + C$ where C is an arbitrary constant and $F(x)$ is any antiderivative of f.

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Example

Find ONE antiderivative, then find the general antiderivative.

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Example

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- $f(x) = x^2 - x + 1$

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Example

Find ONE antiderivative, then find the general antiderivative.

- $f(x) = x^2 - x + 1$
- $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$

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- $f(x) = \sec 2x \tan 2x + \pi$

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Example

Find ONE antiderivative, then find the general antiderivative.

- $f(x) = x^2 - x + 1$
- $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$
- $f(x) = \sec 2x \tan 2x + \pi$
- $f(x) = \frac{3}{2x} - e^{-x}$

L19: Antiderivatives

L19: Antiderivatives

Function

x^n

$\sin kx$

$\cos kx$

$\sec^2 kx$

$\csc^2 kx$

$\sec kx \tan kx$

$\csc kx \cot kx$

L19: ANTIDERIVATIVES

Function	General antiderivative
x^n	$\frac{1}{n+1}x^{n+1} + C$
$\sin kx$	$\frac{-1}{k}\cos kx + C$
$\cos kx$	$\frac{1}{k}\sin kx + C$
$\sec^2 kx$	$\frac{1}{k}\tan kx + C$
$\csc^2 kx$	$\frac{-1}{k}\cot kx + C$
$\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
$\csc kx \cot kx$	$\frac{-1}{k}\csc kx + C$

L19: ANTIDERIVATIVES

L19: Antiderivatives

Function

e^{kx}

$\frac{1}{x}$

$\frac{1}{\sqrt{1-k^2x^2}}$

$\frac{1}{1+k^2x^2}$

$\frac{1}{x\sqrt{k^2x^2-1}}$

a^{kx}

L19: Antiderivatives

Function	General antiderivative
e^{kx}	$\frac{1}{k}e^{kx} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
$\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
$\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C$
a^{kx}	$\frac{1}{k\ln a}a^{kx} + C,\ a > 0,\ a \neq 1$

L19: Antiderivatives

Definition

The *indefinite integral* of f with respect to x is the collection of all antiderivatives of f, denoted

$$\int f(x)\,dx.$$

The symbol $\int$ is the *integral sign*. The function f is the *integrand*, and x is the *variable of integration*.

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- *Constant multiple rule:* $\int kf(x)\,dx = k\int f(x)\,dx.$
- *Negative rule:* $\int -f(x)\,dx = -\int f(x)\,dx.$
- *Sum or Difference rule:*
  $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx.$

A few examples:

Example

What is the slope of the line connecting the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ if f is continuous, $f'(x) = |x|$ and $x_1 = -2$, $x_2 = 3$?

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Example

I leave from home at noon on a bike ride with initial speed 60 mph, and my acceleration for the whole trip is $\frac{dv}{dt} = -30$. When do I get back home? What is my speed at that time?

L20: SIGMA NOTATION

Today- Covered sections: §5.2 (On Exam 2)
Monday- Covered sections: §5.1 (NOT on Exam 2)

No quiz for this lesson.

EXAM 2 (L9-L20) Thursday, October 10

L20: Sigma notation

The notation

$$\sum_{k=0}^{n} a_k = a_0 + a_1 + \cdots + a_n$$

or, add up a_k's from $k = 0$ to $k = n$.
For example,

$$\sum_{k=0}^{3} 2(k+1) = 2(0+1) + 2(1+1) + 2(2+1) + 2(3+1) = 20.$$

L20: Sigma notation

The notation

$$\sum_{k=0}^{\infty} a_k \quad = \quad \lim_{n\to\infty} \sum_{k=0}^{n} a_k.$$

L20: Sigma notation

Expand the sigma notation to compute the sums.

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- $\sum_{i=0}^{4} 2i + 1$
- $\sum_{k=0}^{3} k^2 - 1$

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Write the sum in sigma notation:

- $1 + 2 + 4 + 8 + 16 + \cdots$

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- $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$
- $\frac{-1}{5} + \frac{4}{5} - \frac{9}{5} + \frac{16}{5} - 5 + \frac{36}{5} - \cdots$

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Some rules:

1. Sum rule: $\sum_{k=1}^{n}(a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$.

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If $\sum_{k=1}^{n} a_k = 0$ and $\sum_{k=1}^{n} b_k = 3$, then find

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Example

If $\sum_{k=1}^{n} a_k = 0$ and $\sum_{k=1}^{n} b_k = 3$, then find

- $$\sum_{k=1}^{n}(3a_k - b_k)$$
- $$\sum_{k=1}^{n}(b_k - 1)$$

L20: SIGMA NOTATION

Rule: (The first n squares and cubes)

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

and

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

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Example

Calculate

$$\lim_{n\to\infty} \sum_{k=1}^{n} (\frac{1}{n} - \frac{k^2}{n^2})$$