

Practice Quiz 10 (L30-L32)

1. If a force of 15N stretches a spring 2m past its natural length, how much work does it take to stretch it 5m past its natural length?

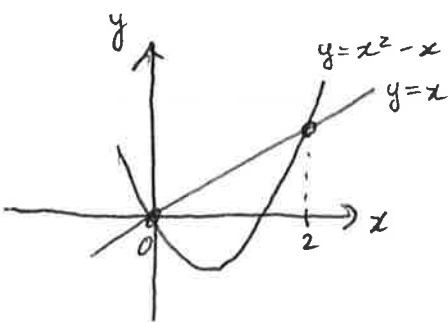
$$F(2) = 15 \quad \& \quad F(x) = kx$$

$$\text{so } k \cdot 2 = 15, \quad k = 7.5$$

$$W = \int_0^5 7.5x \, dx = 3.75x^2 \Big|_0^5 = 25 \cdot 3.75 = 93.75 \text{ N}\cdot\text{m}$$

$$= \boxed{93.75 \text{ joules}}$$

2. Find the center of mass of the region bounded by the parabola $y = x^2 - x$ and the line $y = x$.



$$M_y = \int \tilde{x} \, dm = \int_0^2 x \cdot \delta [f(x) - g(x)] \, dx = \delta \int_0^2 2x^2 - x^3 \, dx = \frac{4\delta}{3}$$

$$M_x = \int \tilde{y} \, dm = \int_0^2 \frac{\delta}{2} [f^2(x) - g^2(x)] \, dx = \frac{\delta}{2} \int_0^2 -x^4 + 2x^2 \, dx = \frac{32\delta}{16}$$

$$M = \int dm = \int_0^2 \delta [f(x) - g(x)] \, dx = \delta \int_0^2 2x - x^2 \, dx = 4\delta/3$$

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M} \right) = \left(1, \frac{8}{5} \right)$$

3. Solve the initial value problem: $\frac{dy}{dt} = ky$ and $y_0 = 5$.

$$\frac{dy}{dt} = ky \Rightarrow \int \frac{1}{y} \, dy = \int k \, dt$$

$$\Rightarrow \ln|y| = kt + C$$

$$\Rightarrow y = e^{kt+C} \quad \& \quad \text{when } t=0 \quad y=5$$

$$\text{so } 5 = e^C$$

$$C = \ln 5$$

$$y = e^{kt + \ln 5}$$

or $\boxed{y = 5e^{kt}}$