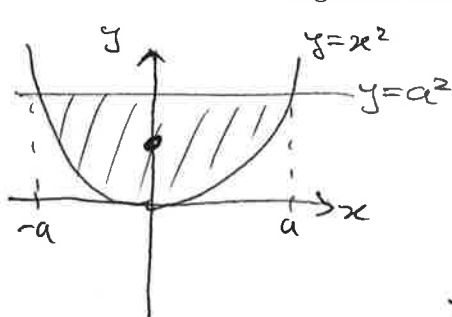## Quiz 10 (L30-L32)

Show all work for full credit.

1. What is the y-value of the center of mass of a plate with constant density in the region R bounded by $y = x^2$ and $y = a^2$, where a is a constant?   (5 pts.)

$y = x^2$
$y = a^2$

$$M_x = \int \breve{y}\, dm = \int_{-a}^{a} \frac{\delta}{2}[a^4 - x^4]\, dx = \delta\left(a^4 x - \tfrac{1}{5}x^5 \big|_0^a\right)$$

$$M_x = \delta\left(a^5 - \tfrac{1}{5}a^5\right) = \frac{4a^5}{5}\delta$$

$$M_y = \int dm = \int_{-a}^{a} \delta[a^2 - x^2]\, dx = 2\delta\left(a^2 x - \tfrac{1}{3}x^3 \big|_0^a\right) = \frac{4}{3}a^2 \cdot \delta$$

$$\bar{y} = \frac{M_x}{M} = \frac{4a^5/5 \cdot \delta}{4a^2/3 \cdot \delta} = \boxed{3a^2/5}$$

2. A spring at rest is 2m in length, and has spring constant $k = \ln 2$. What is the work done to pull the spring from a length of 3m to a length of 5m?   (5 pts.)

$$W = \int_1^3 \ln 2\, x\, dx = \frac{\ln 2}{2} x^2 \Big|_1^3 = \frac{\ln 2}{2}(9-1)$$

$$= \boxed{4 \ln 2 \text{ joules}}$$

3. Which of the following solves the differential equation $y' = y^2$?   (5 pts.)

$$\frac{dy}{dx} = y^2$$

$$\int \frac{1}{y^2}\, dy = \int dx$$

$$\frac{-1}{y} = x + C$$

$$y = \frac{-1}{x + C}$$

(A) $y = \dfrac{1}{x}$

(B) $y = \dfrac{-1}{(x+1)^3}$

(C) $y = \dfrac{1}{(1-x)^2}$

(D) $y = \dfrac{1}{x-1}$

(E) $y = \dfrac{1}{1-x}$

w/ $C = 1$.