Practice Quiz 11 SOLUTIONS (L33-L34)

Show all work for full credit.

1. Compute $\int_1^{e^\pi} \sin(\ln(x))\, dx$. (Hint: do a u-substitution first)

Solution: We set up a u-substitution box $\boxed{\begin{array}{c} w = \ln x \\ dw = 1/x\ dx \end{array}}$ from which it follows that $e^w = x$ and so we have (I'm using w so I can use u later)

$$\int_1^{e^\pi} \sin(\ln(x))\, dx = \int_0^\pi e^w \sin w\, dw.$$

Two integration by parts will allow us to *wrap around* and solve for the integral on the right hand side of the equation above.

$$\boxed{\begin{array}{ll} u = \sin w & dv = e^w\, dw \\ du = \cos w\, dw & v = e^w \end{array}} \qquad \boxed{\begin{array}{ll} u = \cos w & dv = e^w\, dw \\ du = -\sin w\, dw & v = e^w \end{array}}$$

$$\begin{aligned} A &= \int_0^\pi e^w \sin w\, dw \\ &= \sin w \cdot e^w \Big|_0^\pi - \int_0^\pi e^w \cdot \cos w\, dw \\ &= \sin w \cdot e^w \Big|_0^\pi - \left[\cos w \cdot e^w \Big|_0^\pi - \int_0^\pi e^w \cdot (-\sin(w))\, dw\right] \end{aligned}$$

Computing the first two terms of the last line above yield

$$A = 0 - (-e^\pi - 1) - A,$$

so $2A = e^{pi} + 1$, and hence

$$A = \int_0^\pi e^w \sin w\, dw = \boxed{\frac{1}{2}e^\pi + \frac{1}{2}}.$$

2. Which function goes to zero faster, $f(x) = e^{-x}$ or $g(x) = \frac{1}{x}$. (Hint: consider the asymptotic growth of the reciprocal of these functions)

Solution: Its easy to check that $\frac{1}{g} = O(\frac{1}{f})$ and $\frac{1}{f} \neq O(\frac{1}{g})$, so $1/f$ goes to infinity faster than $1/g$, which means that $f(x) = e^{-x}$ goes to zero faster than $g(x) = \frac{1}{x}$.