Quiz 11 (L33-L34)

Show all work for full credit.

1. $\int \ln(x^2)\,dx = 2\int \ln(x)\,dx$

IBP

$u = \ln x \quad dv = dx$
$du = \frac{1}{x}dx \quad v = x$

$= 2\left[x\ln x - \int x\cdot\frac{1}{x}\,dx\right]$

$= 2x\ln x - 2\int 1\,dx$

$= \boxed{2x\ln x - 2x + C}$

2. $\int_0^1 x\sqrt{1-x}\,dx = -\frac{2x}{3}(1-x)^{3/2}\Big|_0^1 - \int_0^1 -\frac{2}{3}(1-x)^{3/2}\,dx$

IBP

$u = x \quad dv = \sqrt{1-x}\,dx$
$du = dx \quad v = -\frac{2}{3}(1-x)^{3/2}$

$= 0 + \frac{2}{3}\cdot\frac{-2}{5}(1-x)^{5/2}\Big|_0^1 = \boxed{\frac{4}{15}}$

also can use:

u-sub

$u = 1-x$
$du = -dx$
$x = 1-u$

$-\int_1^0 (1-u)\sqrt{u}\,du = \int_0^1 (\sqrt{u} - u^{3/2})\,du = \frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\Big|_0^1 = \frac{2}{3} - \frac{2}{5} = \frac{10-6}{15} = \boxed{\frac{4}{15}}$

3. Which grows faster as $n \to \infty$, $f(n) = n$ or $g(n) = (\log_2(n))^2$. Show all your work for full credit.

I show that $g = O(f)$, by showing $\lim_{n\to\infty} \frac{g(n)}{f(n)} \le 1$.

(In fact: $g = o(f)$ since $\lim_{n\to\infty}\frac{g(n)}{f(n)} = 0$).

$\lim_{n\to\infty}\frac{g(n)}{f(n)} = \lim_{n\to\infty}\frac{(\log_2(n))^2}{n} \overset{\text{L'Hop}}{=} \lim_{n\to\infty}\frac{2\log_2(n)\cdot(\log_2(n))'}{1} = \lim_{n\to\infty}\frac{2\log_2(n)}{\ln(2)\cdot n}$

$\overset{\text{L'Hop}}{=} \lim_{n\to\infty}\frac{2\,\frac{1}{\ln(2)\cdot n}}{\ln(2)} = \lim_{n\to\infty}\frac{2}{[\ln(2)]^2\cdot n} = 0$ ✓.

So $g = o(f)$ and $\boxed{g = O(f)}$