

Practice Quiz 4 (L9-L10)

1. Find the derivatives.

(i) $\frac{d}{dx} \left(\frac{1}{x^2} - 6 \sec(3x - 1) + \pi^2 \right)$

Solution: $\frac{-2}{x^3} - 18 \sec(3x - 1) \tan(3x - 1)$

(ii) $f''(x)$ if $f(x) = e^{3x^2-1} - 6x^2 \ln(x)$

Solution: $f'(x) = 6xe^{3x^2-1} - (12x \ln(x) + 6x^2 \frac{1}{x}) = \boxed{6xe^{3x^2-1} - 12x \ln(x) - 6x}$

$f''(x) = 6e^{3x^2-1} + 36x^2e^{3x^2-1} - (12 \ln(x) + 12x \frac{1}{x}) - 6 = \boxed{6e^{3x^2-1} + 36x^2e^{3x^2-1} - 12 \ln(x) - 18}$

2. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

- (i) What is the instantaneous rate of change of the volume of a sphere with respect to its radius?

Solution: $\frac{dV}{dr} = 4\pi r^2$

- (ii) What is the rate of change of the volume of a sphere of radius $r = 2$ with respect to radius.

Solution: $\frac{dV}{dr}(2) = 16$