Practice Quiz 5 (L11-L13)

1. Find the derivatives.

(i) $\frac{d}{dx}\left(\ln\left(\frac{2x-1}{x^2}\right) - 6\tan^{-1}(2x-1)\right)$

Solution: $\frac{d}{dx}\left(\ln(2x-1) - 2\ln(x) - 6\tan^{-1}(2x-1)\right) = \boxed{\dfrac{2}{2x-1} - \dfrac{2}{x} - \dfrac{12}{1+(2x-1)^2}}$

(ii) $\frac{dy}{dx}$ if $ye^{3x} = x + y$

Solution: $\frac{dy}{dx}e^{3x} + 3ye^{3x} = 1 + \frac{dy}{dx}$, then get $\frac{dy}{dx}$ terms to one side. $\frac{dy}{dx}(e^{3x} - 1) = 1 - 3ye^{3x}$, and solve $\frac{dy}{dx} = \boxed{\dfrac{1-3ye^{3x}}{e^{3x-1}}}$.

2. A 12-ft ladder is leaning against the wall as the base starts to slide away. By the time the base is 8 ft from the wall, the base is moving at the rate of 4 ft/sec.

(i) How fast is the side of the ladder sliding down the wall at this time?

Solution: Draw a picture. Given: base distance $x = 8$, $\frac{dx}{dt} = 4$ and $\sqrt{x^2+y^2} = 12$. Take $\frac{d}{dt}$ to both sides of equation. $\frac{d}{dt}(\sqrt{x^2+y^2}) = \frac{d}{dt}12$. Solve for $\frac{dy}{dt}$. $\frac{2x\frac{dx}{dt}+2y\frac{dy}{dt}}{2\sqrt{x^2+y^2}} = 0$ implies $\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$. When $x = 8$ then $y = 4\sqrt{5}$. So $\boxed{\dfrac{dy}{dt} = -8\sqrt{5}/5}$.

(ii) At what rate is the area formed by the triangle formed by the ladder, ground, and wall changing at this time?

Solution: $A = \frac{1}{2}xy$, so $\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$. We already found that when $x = 8$ we have $\frac{dx}{dt} = 4$, $y = 4\sqrt{5}$ and $\frac{dy}{dt} = 8\sqrt{5}5$. Plug in: $\frac{dA}{dt} = \frac{1}{2}(4)(4\sqrt{5}) + \frac{1}{2}(8)(-8\sqrt{5}/5) = \boxed{8\sqrt{5}/5}$.

(iii) At what rate is the angle formed by the ladder and the ground changing at this time?

Solution: $\theta = \tan^{-1}(x/y)$. Chain rule $\frac{d\theta}{dt} = \frac{1}{1+(\tan^{-1}(x'y))^2} \cdot \frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2} = \frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2+x^2}$. Plug in: $\frac{d\theta}{dt} = \boxed{\dfrac{13\sqrt{5}}{45}}$.