

Quiz 5 (L11-L13)

Show all work for full credit.

1. Find the derivatives.

(4 pts. each)

(i) $\frac{dy}{dx}$ if $x^3 - xy^2 + 3y = 10$.

$$3x^2 - x \cdot 2y \frac{dy}{dx} - y^2 + 3 \frac{dy}{dx} = 0$$

$$3x^2 - y^2 = \frac{dy}{dx} (2xy - 3)$$

$$\frac{dy}{dx} = \frac{3x^2 - y^2}{2xy - 3}$$

(ii) $f'(x)$ if $f(x) = \ln\left(\frac{x^2+1}{x^2e^x}\right) - \frac{x-1}{1-x} \tan^{-1}(1-x)$

$$f(x) = \ln(x^2+1) - 2\ln(x) - x + \tan^{-1}(1-x)$$

$$f'(x) = \frac{2x}{x^2+1} - \frac{2}{x} - 1 + \frac{-1}{1+(1-x)^2}$$

2. The surface area of a sphere of radius r is $A = 4\pi r^2$.(i) Find $\frac{dA}{dt}$ the change of area with respect to time.

(4 pts.)

$$\frac{d}{dt} A = \frac{d}{dt} 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

(ii) How fast is the surface area changing with respect to time if the radius is changing at .5 meters per second.

(3 pts.)

$$\frac{dA}{dt} = 8\pi r (.5) = 4\pi r \quad \text{when} \quad \frac{dr}{dt} = .5$$