Math 1501 | **Calculus I** | **Fall '13**

Practice Quiz 6 (L14-L16)

1. What is the linearization of $f(x) = \sqrt{x}$?

$$L(x) = f(a) + f'(a)(x-a)$$

$$L(x) = \sqrt{a} + \frac{1}{2\sqrt{a}}(x-a)$$

(i) Use calculus to approximate $\sqrt{82}$.

$a = 81$, $\sqrt{a} = 9$

$$\sqrt{82} \approx L(82) = \sqrt{81} + \frac{1}{2\sqrt{81}}(82-81)$$

$$= 9 + \frac{1}{18}(1) = \boxed{163/18}$$

2. What are the critical points of $f(x) = \ln(x^2 - 2x)$?

$$f'(x) = \frac{2x-2}{x^2-2x} = 0 \quad \text{if } x = 1, \text{ undefined if } x = 0, 2.$$

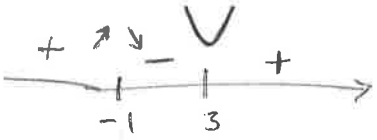
$x = 1$: not in domain either. $x = 0, 2$: not in the domain!

No critical points.

~~Then critical point of f only at $x = 1$~~

3. Suppose $f'(x) = \frac{x-3}{x+1}$. Find and classify the critical points of f as local max, local min, or neither.

$$f'(x) = \frac{x-3}{x+1} = 0 \quad \text{if } x = 3, \text{ undefined at } x = -1.$$



$x = 3$ is a local minimum

$x = -1$ is { a local maximum if $x = -1$ is in the domain of f; neither, not a critical point if $x = -1$ is NOT in the domain of f.

4. Find the absolute max and absolute min of $f(x) = \sin^{-1}(\cos(3x))$ on the interval $[0, \frac{\pi}{2}]$.

f' not defined when $\cos^2(3x) = 1$, $\cos(3x) = \pm 1 \to 3x = 0, \pi$, $x = 0, \pi/3$

$$f'(x) = \frac{1}{\sqrt{1-\cos^2(3x)}} * (-\sin(3x)) * 3$$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$x = 0, \pi/3$: critical points

$$= \frac{-3\sin(3x)}{\sqrt{\sin^2(3x)}} = \frac{-3\sin(3x)}{\pm\sin(3x)} = \pm 3 \quad \text{(never zero)}.$$

in particular, no c's s.t. $f'(c) = 0$ in the domain.

$$f(\pi/3) = \sin^{-1}(\cos(3 \cdot \tfrac{\pi}{3})) = \sin^{-1}(-1) = -\pi/2$$

$$f(0) = \sin^{-1}(\cos(0)) = \sin^{-1}(1) = \pi/2$$

$$f(\pi/2) = \sin^{-1}(\cos(3\pi/2)) = \sin^{-1}(0) = 0$$

ABSOLUTE MAX VALUE of $\pi/2$ @ $x = 0$

ABSOLUTE MIN VALUE of $-\pi/2$ @ $\pi/3$