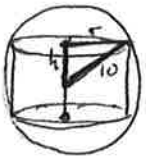
Practice Quiz 7 (L17-L19)

1. Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. What is the maximum volume?



$V = \pi r^2(2h)$ maximize
$r^2 + h^2 = 100$
$r^2 = 100 - h^2$

$V = \pi(100 - h^2)(2h)$
$V = 200\pi h - 2\pi h^3$
$V' = 200\pi - 6\pi h^2 \leq 0$

$h^2 = \frac{200}{6} = \frac{100}{3}$ $h = \pm\sqrt{\frac{100}{3}}$ ← pick positive one.

Dimensions of cylinder: $20\sqrt{3}/3 \times \sqrt{\frac{2}{3}}\cdot 10$ | MAX VOLUME ≈ 2418.4

$h = 10\sqrt{3}/3$
$r = \sqrt{100 - (10\sqrt{3}/3)^2} = \sqrt{\frac{900}{9} - \frac{100\cdot 3}{9}} = \sqrt{\frac{600}{9}} = \sqrt{\frac{200}{3}}$
$= \sqrt{\frac{2}{3}}\cdot 10$

2. What is the general antiderivative of $\frac{1}{x} - \ln(2) + e^{-x}$.

$\ln|x| - \ln(2)\cdot x - e^{-x} + C$

3. Let $f(x) = \frac{x^2-3}{x-2}$. Find the intervals where the function $f(x)$ is increasing, decreasing, concave up, and concave down. Find the critical points and inflection points of f, and any horizontal or vertical asymptotes. Sketch $y = f(x)$.

$f'(x) = \frac{(x-2)(2x) - (x^2-3)(1)}{(x-2)^2} = 0$ if $2x^2 - 4x - x^2 + 3 = 0$

$f'(x) = \frac{x^2-4x+3}{(x-2)^2}$

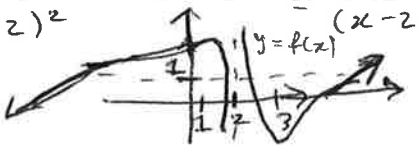
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = 3, 1$

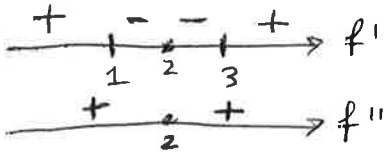
$f''(x) = \frac{(x-2)(2x-4) - (x^2-4x+3)(2(x-2)(1))}{(x-2)^4}$

$= 0$ if $2x^2 - 4x - 4x + 8 - 2x^2 + 8x - 6x + 12 = 0$
$-2x^3 + 14x^2 - 18x + 20 = 0$

$f''(x) = \frac{(x-2)(2x-4) - 2x^2 + 8x - 6}{(x-2)^3}$

$f''(x) = \frac{2x^2 - 4x - 4x + 8 - 2x^2 + 8x - 6}{(x-2)^3} = \frac{2}{(x-2)^2}$





no HA @ ~~y=1~~ VA @ $x = 2$

no inflection pts
$x = 1$ MAX; $x = 3$ MIN