Practice Quiz 8 (L24-L26)

1. Find the general antiderivative. $\int x^3(x^2+1)^{99}\,dx.$ $= \int \frac{1}{2}(u-1)\,u^{99}\,du$

$u = x^2+1 \quad x^2 = u-1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int u^{100} - u^{99}\,du$

$= \boxed{\frac{(x^2+1)^{101}}{202} - \frac{(x^2+1)^{100}}{200} + C}$

2. What is the area of the region bounded by the curves $y = \frac{1}{2}\sec^2 x$, $y = -4\sin^2 x$, $x = -\pi/3$ and $x = \pi/3$?

There are no intersection points.

$$\text{Area} = \int_{-\pi/3}^{\pi/3} \frac{1}{2}\sec^2 x + 4\sin^2 x\,dx$$

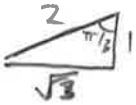
$\sin^2(x) = \frac{1}{2}(1-\cos 2x)$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} \sec^2 x\,dx + 2\int_{-\pi/3}^{\pi/3}(1-\cos(2x))\,dx$$

$$= \frac{1}{2}\tan x\Big|_{-\pi/3}^{\pi/3} + 2x - \sin 2x\Big|_{-\pi/3}^{\pi/3}$$

$$= \sqrt{3} + \frac{4\pi}{3} - \sqrt{3} = \boxed{4\pi/3}$$

$\sin(2\pi/3) = \sin(\pi/3) = \sqrt{3}/2$



3. Suppose $\int_0^1 f(x)\,dx = 3$. Find $\int_{-1}^0 f(x)\,dx$ if f is even. What is the value of the latter definite integral if f is odd?

f even $\rightarrow$ symmetric about y-axis $\rightarrow$ $f(x) = f(-x)$

f even $\rightarrow$ $\int_{-1}^{0} f(x)\,dx = \boxed{3}$

f odd $\rightarrow$ $f(x) = -f(-x)$.

$\rightarrow \int_{-1}^{0} f(x)\,dx = \boxed{-3}$