

Quiz 8 (L24-L26)

Show all work for full credit.

1. Find the value. $\int_{-1}^1 \frac{x}{2-x} dx = \int_3^1 (2-u)u^{-1}(-1) du$ (5 pts.)

$$u = 2 - x \quad x = 2 - u$$

$$du = -dx$$

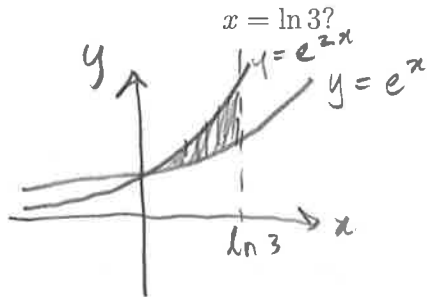
$$x = -1 \Rightarrow u = 3 \quad x = 1 \Rightarrow u = 1$$

$$= \int_1^3 \frac{2}{u} - 1 du = 2 \ln u - u \Big|_1^3$$

$$= 2 \ln 3 - 3 - (2 \ln 1 - 1)$$

$$= \boxed{2 \ln 3 - 2}$$

2. What is the area of the triangular region bounded by $y = e^x$, $y = e^{2x}$ and the line $x = \ln 3$? (5 pts.)



$$\int_0^{\ln 3} e^{2x} - e^x dx = \frac{1}{2} e^{2x} - e^x \Big|_0^{\ln 3}$$

$$= \frac{1}{2} e^{2 \ln 3} - e^{\ln 3} - \left(\frac{1}{2} - 1 \right)$$

$$= \frac{9}{2} - 3 + \frac{1}{2} = \boxed{2}$$

3. Which definite integral represents the area under $y = \cot x$ and above the x -axis over the interval $(\pi/4, \pi/2)$ after a u -substitution? (5 pts.)

(A) $\int_0^1 \frac{-1}{u} du$

(B) $\int_{\sqrt{2}/2}^1 \frac{-1}{u} du$

(C) $\int_0^{\sqrt{2}/2} \frac{1}{u} du$

(D) $\int_{\sqrt{2}/2}^1 \frac{1}{u} du$

$$\int_{\pi/4}^{\pi/2} \cot x dx = \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx$$

$$= \int_{\sqrt{2}/2}^1 \frac{1}{u} du$$



$$u = \sin x$$

$$du = \cos x dx$$

$$x = \pi/4 \rightarrow u = \sqrt{2}/2$$

$$x = \pi/2 \rightarrow u = 1$$