# Practice Quiz 9 (L27-L29)

1. Find the volume of the solid which lies between the planes perpendicular to the $x$-axis at $x = 1$ and $x = -1$, whose cross sections perpendicular to the $x$-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. (See page 371 problem 2)

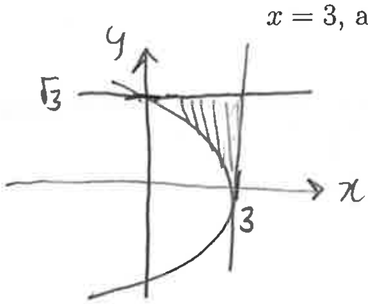
$$A(x) = \pi\left(\tfrac{1}{2}(2 - x^2 - x^2)\right)^2 = \pi(1 - x^2)^2$$

$$\boxed{\frac{16\pi}{15}}$$

$$V = \int_{-1}^{1} \pi(1-x^2)^2\, dx = \pi \int_{-1}^{1} (1 - 2x^2 + x^4)\, dx$$

$$= \pi\left(x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5 \Big|_{-1}^{1}\right) = 2\pi\left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right) = 2\pi\left(\frac{15 - 10 + 3}{15}\right)$$

2. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{3}$, $x = 3$, and $x = 3 - y^2$ across the $x$-axis. (See page 379 problem 4)




$x = 3 - y^2$ $\quad y = \sqrt{3 - x}$

Washer $$\int_0^3 \pi\left(\sqrt{3}^2 - \left(\sqrt{3-x}\right)^2\right) dx$$ $$\boxed{9\pi/2}$$

$$= \int_0^3 \pi(3 - 3 + x)\, dx = \pi \int_0^3 x\, dx$$

$$= \pi/2 * x^2 \Big|_0^3$$

3. Find the surface area of the surface obtained by revolving the curve segment $y = x^3$, $0 \le x \le 2$, about the $x$-axis.

$$SA = \int_0^2 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 9x^4$

$$= \int_0^2 2\pi x^3 \sqrt{1 + 9x^4}\, dx$$

$u = 9x^4$

$du = 36x^3\, dx$

$$= \frac{2\pi}{36} \int_*^* u^{1/2}\, du = \frac{\pi}{9\,\cancel{18}} \cdot \frac{\cancel{2}}{3} u^{3/2} \Big|_*^* = \frac{\pi}{27}(1 + 9x^4)\Big|_0^2$$

$$= \pi/27\,(16 * 9) = \boxed{16\pi/3}$$