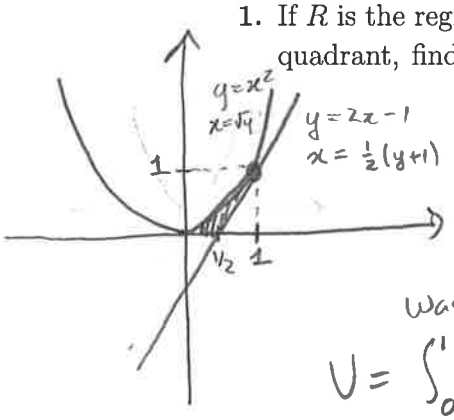
Quiz 9 (L27-L29)

Show all work for full credit.

1. If R is the region bounded by the curve $y = x^2$ and the line $y = 2x - 1$ in the first quadrant, find the volume of the solid obtained by rotation R about the y-axis. (5 pts.)



$x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$R(y) = \frac{1}{2}(y+1)$

$r(y) = \sqrt{y}$

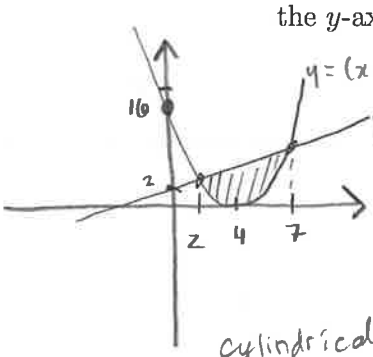
$= \frac{\pi}{4}\left(\frac{1}{3}y^3 - y^2 + y\Big|_0^1\right)$

$= \frac{\pi}{4}\left(\frac{1}{3} - 1 + 1\right) = \boxed{\pi/12}$

Washer method

$$V = \int_0^1 \pi\left(R(y)^2 - r(y)^2\right) dy = \pi\int_0^1 \left[\frac{1}{4}(y^2+2y+1) - y\right] dy = \frac{\pi}{4}\int_0^1 y^2 - 2y + 1 \, dy$$

2. Let R be the region bounded by $y = (x-4)^2$ and $y = x + 2$. Set up and simplify the integral which computes the area of the solid obtained by rotating R about the y-axis. Do NOT integrate. (5 pts.)



$(x-4)^2 = x + 2$

$x^2 - 8x + 16 - x - 2 = 0$

$x^2 - 9x + 14 = 0$

$(x-7)(x-2) = 0$

$x = 7, 2$

$r(x) = x$

$h(x) = x + 2 - (x-4)^2$

$= 2\pi\int_2^7 x\left[x + 2 - x^2 + 8x - 16\right] dx$

$= \boxed{2\pi\int_2^7 (-x^3 + 9x^2 - 14x)\, dx}$

cylindrical shell method

$$V = \int_2^7 2\pi\, r(x)\, h(x)\, dx = 2\pi\int_2^7 x\left[x + 2 - (x-4)^2\right] dx$$

3. Which integral computes the length of the curve $y = x^2$, $0 \le x \le 1$? (5 pts.)

Arc length $= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$\frac{dy}{dx} = 2x$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^2$

(A) $\int_0^1 \sqrt{1+x^4}\, dx$

(B) $\int_0^1 \sqrt{1+2x^2}\, dx$

(C) $\int_0^1 \sqrt{1+4x^2}\, dx$

(D) $\int_0^1 \sqrt{1+x^2}\, dx$