Math 1501 Calc I

Summer 2015

QUP SOUP w/ GTcourses

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§3.3 & §3.5: Differentiation rules

Covered sections: §3.3 & §3.5

Exam 1 (Ch.1 - Ch.3) Thursday, May 28

§3.3 & §3.5: DIFFERENTIATION RULES

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- the slope of the tangent line of $y = f(x)$ at x,
- the instantaneous rate of change of $f(x)$ at x,
- defined as the limit of the difference quotient of $f(x)$ near x.

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Rule: (Power rule)

$\frac{d}{dx}(x^n) = nx^{n-1}$ *for any* n.

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Rule: (Constant multiple & Sum rules)

$$\frac{d}{dx}(cf) = c\frac{df}{dx} \qquad \frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx},$$

for any constant c *and functions* $f(x)$, $g(x)$.

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Rule: (Quotient rule)

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

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§3.3 & §3.5: Differentiation rules

Definition
The derivative of the derivative of f is called the *second derivative* and denoted f''. The n-th derivative of f is the function obtained by taking the derivative of f n-times, and denoted $f^{(n)}$ if $n > 3$.

$$f, f', f'', f''', f^{(4)}, \ldots, f^{(n)}, \ldots$$

The derivatives of trig functions:

$$(\sin\theta)' = \cos(\theta) \qquad (\csc\theta)' = -\csc(\theta)\cot(\theta)$$

$$(\cos\theta)' = -\sin(\theta) \qquad (\sec\theta)' = \sec(\theta)\tan(\theta)$$

$$(\tan\theta)' = \sec^2(\theta) \qquad (\cot\theta)' = -\csc^2(\theta)$$

§3.4 & §3.6: THE CHAIN RULE & APPLICATIONS OF THE DERIVATIVE

Covered sections: §3.4 & §3.6

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§3.4 & §3.6: The chain rule & applications of the derivative

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In Economics:

- If $c(x)$ is the cost to produce x units of a product, then c' is the marginal cost of production.

§3.4 & §3.6: THE CHAIN RULE & APPLICATIONS OF THE DERIVATIVE

Rule: (The chain rule)

The derivative of $g \circ f(x)$ is

$$f'(x) \cdot g'(f(x)),$$

or using alternate notation

$$\frac{df}{dx}(x) \cdot \frac{dg}{dx}(f(x)).$$