Worksheet 4: Chapter 2/3 (ε-δ defn, limits of the secant/DQ)

1. From the definition of

$$\lim_{x\to 3} x^2 = 9$$

Find the **largest** $\delta > 0$ such that if $|x-3| < \delta$, then $|x^2 - 9| < \varepsilon$, where $\varepsilon = 2$.

2. Let $f(x) = \sqrt{x}$. Find

$$L = \lim_{x\to 16} f(x).$$

Then, find the largest $\delta > 0$ such that

$$|x - 16| < \delta \quad \implies \quad |f(x) - L| < \varepsilon$$

for $\varepsilon = 4$.

3. Consider the function $f(x) = \frac{1}{x}$, and we will examine a **secant line** of $f(x)$ as well as $f'(2)$ which is the derivative of f at $x = 2$, the limit of the slopes of the secant lines near $x = 2$:

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.$$

(a) Find the slope of the secant line of the points $(2, 1/2)$ and $(4, 1/4)$.

(b) Guess the limit of the slopes of the secant lines at $x = 2$ by either creating a table (in a spreadsheet) or using graphing software (like GeoGebra).

(c) Find the *instantaneous rate of change* at $x = 2$. That is, find

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.$$

Note that "instantaneous rate of change" and "limit of the slopes of the secant lines" and "the derivative" and "limit of the difference quotient" are all (essentially) synonyms.

4. On a walk with my dog my distance from home after t minutes is given by

$$f(t) = t^3/8 - t^2 + 2t.$$

What is my average speed in the first 30 seconds of my walk? *(1)* What is my average speed in the first 2 minutes? *(2)* What is the *average rate of change* of the function $f(x)$ on the interval $[0, 4]$ and *(3)* how can you phrase this question to be similarly phrased as the previous two? *(4)* Set up but **do not solve** a limit which could be evaluated to find $f'(2)$, and finally *(5)* interpret the meaning of $f'(2)$ in this example.