Worksheet 6: Chapter 3 (Implicit differentiation and log derivatives)

1. Find the derivative $f'(x)$ two ways: *(first way)* first use properties of logs to simplify and then take the derivative, and *(second way)* take the derivative directly using the chain rule. Which way is easier for you?

$$f(x) = \ln\left(\frac{x^2}{1-x}\right)$$

2. Find the equation of the one of the lines tangent to the curve given by the equation

$$3xy - y^2 - x^3 = 1$$

at a point on the curve where $x = -2$. *There are two possible answers!* Illustrate the problem using software or sketch the situation.

3. Show that the point $(2, 4)$ lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal to the curve at the point $(2, 4)$. Illustrate the example using software or make a sketch.

4. Use logarithmic differentiation to find the derivative of y with respect to t if

$$y = \left(\sqrt{t}\right)^t.$$

5. Use either a 30-60-90 or a 45-45-90 reference triangle to evaluate the given expression.

(a) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(b) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

6. Use the graphs of the trig functions to evaluate the limits. If the limit does not exist write either DNE, $+\infty$ DNE, or $-\infty$ DNE whichever is the most appropriate.

(a) $\lim_{x\to 1} \sin^{-1} x$

(b) $\lim_{x\to\infty} \sec^{-1}(x)$

(c) $\lim_{x\to 1^-} \cos^{-1}(x)$

7. Use the inverse trig derivative identities to find y'.

(a) $y = \cos^{-1}(x^2)$

(b) $y = \sec^{-1}(5x)$