Worksheet 7: Chapter 3 (Related rates)

1. Water runs into a conical tank at the rate of 9 ft³/min. The tank stands pointed downwards and has a height of 10 feet and the conical base radius is 5ft. How fast is the water rising when the water is 3 ft deep? Recall: the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.

2. A balloon rises above a stationary point on the ground 500 ft from an observer. At a certain time the observers' viewing angle is $\pi/4$, and the angle is increasing at a rate of 0.14 rad/min. At this time, how high off the ground is the balloon? how fast is the balloon rising?

3.	A 13-ft ladder is leaning against the wall of a house when its base starts to slip and slides away from the wall. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec.
	(a) At this moment, how fast is the side of the ladder sliding down the wall?
	(b) At what rate is the area of the triangle formed by the ladder, the ground, and the wall changing at this moment?
	(c) At what rate is the angle θ made by the ground and the ladder changing at that moment?

- 4. The radius r and height h of a right circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$. This problem asks you to analyze how the volume of the cylinder changes if the radius r or the height h change over time.
 - (a) Suppose r is a constant so that dr/dt = 0. Relate dV/dt and dh/dt using the volume equation $V = \pi r^2 h$ and differentiation with respect to the time variable t.

(b) Suppose now that h is constant, so that dr/dt, but that the radius r = r(t) of the cylinder changes over time. Find an equation relating dV/dt to dr/dt.

(c) Finally, find an equation which relates dV/dt to both dr/dt and dh/dt. Do not assume that either r or h is constant.