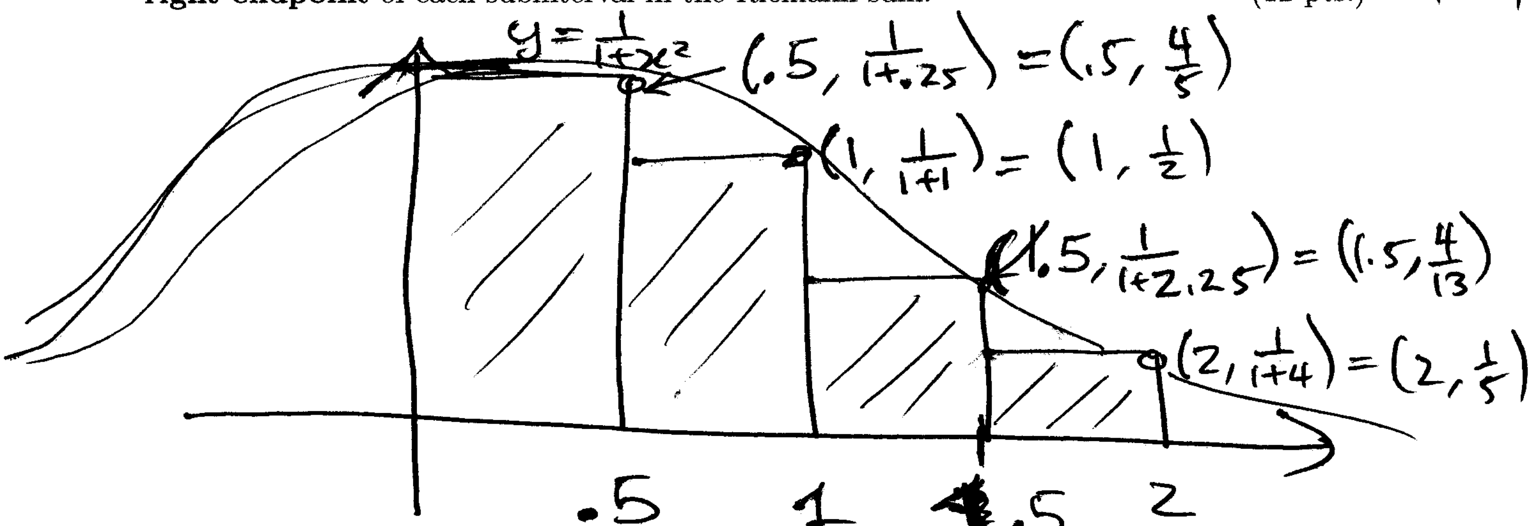
EXAM 1

$1+\frac{1}{4}=\frac{5}{4}$

1. Approximate the area under the curve $y=\frac{1}{1+x^2}$ over the interval $[0,2]$ using a Riemann sum approximation with $n=4$ rectangles and using a *lower sum estimate*, that is, use the **right endpoint** of each subinterval in the Riemann sum. (12 pts.)

$1+\frac{9}{4}=\frac{13}{4}$



Area $=\Delta x \sum$ heights

$=\frac{1}{2}\left(\frac{4}{5}+\frac{1}{2}+\frac{4}{13}+\frac{1}{5}\right)$

$=\frac{1}{2}\left(\frac{146+65+40+26}{130}\right)=\boxed{\frac{277}{260}}$

105

$5*2*13=130$

$4*2*13=26*4=146$

$131+146=277$

2. Find an upper bound on the error $|E_T|$ of a finite sum approximation of $\int_0^2 \frac{1}{1+2x}\,dx$ using $n=4$ trapezoids. You may assume that (8 pts.)

$$|E_T| \le \frac{(b-a)^3 M}{12n^2}, \text{ if } |f''(x)| \le M \text{ on } [0,2].$$

Using the formula

$f(x)=\frac{1}{1+2x}$

$f'(x)=\frac{-1}{(1+2x)^2}*2$

$f''(x)=\frac{2}{(1+2x)^3}*2*2$

$M=8$

$(b-a)^3=2^3=8$

$12n^2=12\cdot 4^2=12*16$

$|E_T|\le \frac{(b-a)^3 M}{12n^2}=\frac{\cancel{8}\cancel{8}}{3\cancel{12}*\cancel{16}}=\boxed{\frac{1}{3}}$

$f''(x)=\frac{8}{(1+2x)^3}$ which is max of $\frac{8}{(1+2\cdot 0)^3}=8$ on $[0,2]$ since $\frac{\text{const}}{\text{Big}}=$ sma

3. Find the general anti-derivative. (6 pts. each)

(a) $\int \sec(3x)\tan(3x)\,dx$

$$\int \frac{1}{\cos(3x)} * \frac{\sin(3x)}{\cos(3x)}\,dx = \int \frac{1}{\cos^2(3x)} * \sin 3x\,dx$$

$$= \int \frac{1}{u^2}\,du = \frac{-1}{u} + C = \boxed{\frac{-1}{\cos(3x)} + C}$$

(b) $\int \cos(\pi x) - e^{-x/2} + \pi^3\,dx$

$$\frac{1}{\pi}\sin(\pi x) + 2e^{x/2} + \pi^3 x + C$$

(c) $\int \frac{x-1}{x}\,dx$

$$= \int \left(\frac{x}{x} - \frac{1}{x}\right)dx$$

$$= \int 1 - \frac{1}{x}\,dx = x - \ln x + C$$

(d) $\int p^x\,dx, \quad p > 0.$

$y = p^x$

$\ln y = x\ln(p)$

$(\ln y)' = (x\ln p)' = \ln p$ constant p

$\frac{y'}{y} = \ln p \Rightarrow y' = y\ln p$

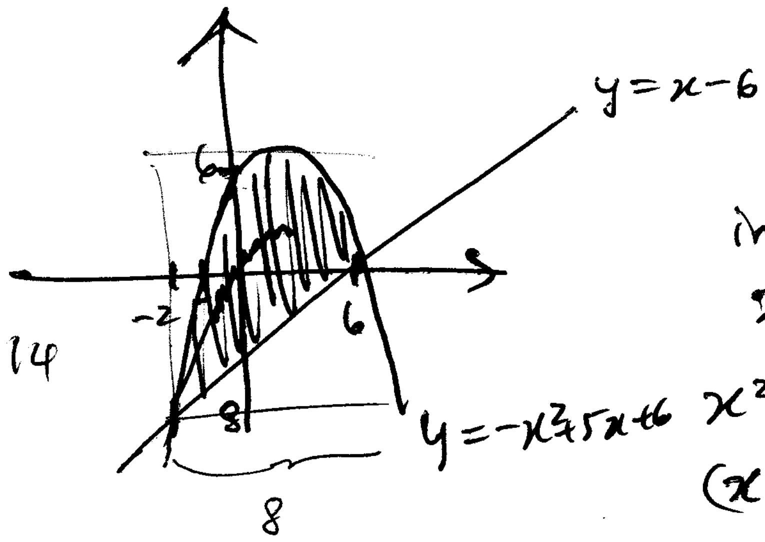
$$\int p^x = \boxed{\frac{1}{\ln p} p^x + C}$$

FTC

Since $(p^x)' = p^x \ln p$

$14 * 8 = 112$ upper estimate

4. Find the area bounded by the line $y = x - 6$ and the curve $y = -x^2 + 5x + 6$. (16 pts.)



$-x^2+5x+6 = -(x^2-5x-6)$

$= -(x-6)(x+1)$

$=0$ when $x=6,-1$

intersection pts $y=y$

$x-6 = -x^2+5x+6$

$x^2-4x-12=0$

$(x-6)(x+2)=0 \quad x=6,-2$

$A=\int$ top-bot $= \int_{-2}^{6} -(x^2-4x-12)\,dx = \left(-\frac{x^3}{3}+2x^2+12x \Big|_{-2}^{6}\right)$

$= \left[-\frac{6^3}{3}+2(6)^2+12(6)\right] - \left[\frac{-2^3}{3}+2\cdot 4+12\cdot 2\right]$

5. Find $g'(x)$ if (8 pts.)

$$g(x) = \int_{1}^{x^2-x} \sin(3t^2)\,dt.$$

$$\boxed{g'(x) = \sin(3(x^2-x)^2) * (2x-1)}$$

$\cancel{\frac{-6^2 * 6}{3}} + \cancel{2\cdot 6^2} + 12\cdot 6$

$+\frac{8}{3} - 8 - 24$

$= 12\cdot 6 - 12\cdot 2 + \frac{8}{3} - 8$

$= 12 * 4 + \frac{8}{3} - 8$

$= 48 - 8 + \frac{8}{3}$

$= 40 + 8/3 = \boxed{128/3}$ whew!

$\cancel{=216+16+\frac{8}{3}}$

$\cancel{=232+8/3}$

$= \cancel{\frac{696+8}{3}} = \cancel{\frac{704}{3}}$

6. Integrate. (8 pts. each)

(a) $\displaystyle\int \frac{2e^{\sqrt{1-x}}}{\sqrt{1-x}}\,dx$

$$= -4\int e^{\sqrt{1-x}} \frac{-1}{2\sqrt{1-x}}\,dx$$

$u = \sqrt{1-x}$

$du = \frac{-1}{2\sqrt{1-x}}\,dx$

$$= -4\int e^u\,du$$

$$= -4e^u + C$$

$$\boxed{= -4e^{\sqrt{1-x}} + C}$$

(b) $\displaystyle\int_0^{\sqrt{\pi}/2} \sqrt{3}x\sec^2(x^2)\,dx$

~~IBP~~ u-sub.

$u = x^2$

$du = 2x\,dx$

~~$u = x \quad dv = \sec^2(x^2$~~

~~$du = dx$~~

$$\int_0^{\sqrt{\pi}/2} \sqrt{3}\,x\sec^2(x^2)\,dx = \frac{\sqrt{3}}{2}\int_*^* \sec^2(u)\,du$$

$$= \frac{\sqrt{3}}{2}\tan(u)\Big|_*^*$$

$$= \frac{\sqrt{3}}{2}\tan(x^2)\Big|_0^{\sqrt{\pi}/2}$$

$$= \frac{\sqrt{3}}{2}\left(\tan(\pi/4) - 0\right)$$

$$= \boxed{\sqrt{3}/2}$$

(c) $\displaystyle\int_1^{e^2} \frac{\ln(x)}{8x}\,dx$

$$= \frac{1}{8}\int_*^* u\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \frac{1}{8}u^2\Big|_*^*$$

$$= \frac{1}{8}(\ln(x))^2\Big|_1^{e^2}$$

$$= \frac{1}{8}\left((\ln(e^2))^2 - 0\right)$$

$$= \frac{1}{8}(2)^2 = \frac{4^2}{8} = \frac{16}{8} = \boxed{2}$$