1. Suppose the amount of oil pumped from a well decreases continuously at a rate of 8% per year. When will the well's output fall below 1/4 of its present value? (10 pts.)

$$\frac{dP}{dt} = -0.08P$$

$$\frac{dP}{dt} = -\frac{8}{100}P$$

$$\int \frac{1}{P}\,dP = \int \frac{-8}{100}\,dt$$

$$\ln(P) = \frac{-8}{100}t + C$$

$$P = Ce^{-8/100\,t}$$

Set $C = 100$ for 100%

$$P(t) = 100e^{-8/100\,t}$$

Find t s.t.

$$P(t) = 25 = 100e^{-8/100\,t}$$

$$\frac{1}{4} = e^{-8/100\,t}$$

$$\ln\left(\tfrac{1}{4}\right) = \frac{-8}{100}t$$

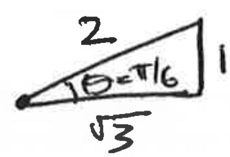
✓

$$t = \boxed{\ln(2) * 25} \approx 17.328$$

2. Find the exact value. Simplify your answer for full credit. (10 pts.)

$$\int_0^{3/2} \frac{2}{\sqrt{9 - x^2}}\, dx$$

$$2\int_0^{3/2} \frac{1}{\sqrt{9-x^2}}\,dx = \frac{2}{3}\sin^{-1}(3x)\Big|_0^{3/2}$$



$$= \frac{2}{3}\sin^{-1}\left(\tfrac{1}{2}\right) - 0$$

$$= \frac{2}{3}\cdot \pi/6 = \boxed{\pi/9}$$

3. Integrate. (8 pts. each)

(a) $\int 4x \sec^2(2x)\, dx$ $= 2x\tan(2x) - \int 2\tan(2x)\, dx$

IBP

$$\boxed{\begin{aligned} u &= 4x & dv &= \sec^2(2x)\,dx \\ du &= 4\,dx & v &= \tfrac{1}{2}\tan(2x) \end{aligned}}$$

$= \boxed{2x\tan(2x) - \ln|\cos(2x)| + C}$

(b) $\int x^2 \ln(x^3)\, dx$ $= \frac{1}{3}\int \ln(u)\, du$

u-sub

$$\boxed{\begin{aligned} u &= x^3 \\ du &= 3x^2\,dx \end{aligned}}$$

$= \frac{1}{3}[u\ln(u) - u] + C$

$= \boxed{\frac{1}{3}x^3\ln(x^3) - x^3 + C}$

(c) $\int 3\cos^3(x)\, dx$ $= \int 3\cos^2 x \cos x\, dx$

$= \int 3(1 - \sin^2 x)\cos x\, dx$

u-sub

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$= \int (3 - 3u^2)\, du$

$= 3u - u^3 + C = \boxed{3\sin x - \sin^3 x + C}$

4. Integrate. *Hint: a trigonometric substitution first will give you a trigonometric integral.*

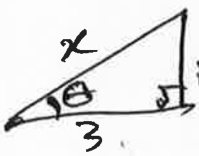
(10 pts.)

$$\int \frac{x^3}{x^2-9}\,dx$$

trig-sub

$$\boxed{\begin{aligned} x &= 3\sec\theta \\ dx &= 3\sec\theta\tan\theta\,d\theta \end{aligned}}$$

$$= \int \frac{27\sec^3\theta}{\cancel{3\tan\theta}}\cdot \cancel{3}\sec\theta\,\cancel{\tan\theta}\,d\theta$$



$\sec\theta = \frac{x}{3} = \frac{hyp}{adj}$

$\tan\theta = \frac{\sqrt{x^2-9}}{3} = \frac{opp}{adj}$

$$= \int 27\sec^4\theta\,d\theta$$

$$= \int 27(\sec^2\theta)\sec^2\theta\,d\theta$$

u-sub

$$\boxed{\begin{aligned} u &= \tan\theta \\ du &= \sec^2\theta\,d\theta \end{aligned}}$$

$$= \int 27(1+\tan^2\theta)\sec^2\theta\,d\theta$$

$$= \int 27(1+u^2)\,du$$

$$= 27u + 9u^3 + C = 27\tan\theta + 9\tan^3\theta + C$$

$$= \boxed{9\sqrt{x^2-9} + \frac{1}{3}(x^2-9)^{3/2} + C}$$

5. Does the improper integral $\int_1^\infty \frac{1}{x^{3/2}}\,dx$ converge or diverge? If it converges, find the value.

(6 pts.)

$\boxed{\text{Converges,}}$

$$\int_1^\infty \frac{1}{x^{3/2}}\,dx = \int_1^\infty x^{-3/2}\,dx = -2x^{-1/2}\Big|_1^\infty = 0-(-2) = \boxed{2}$$

(a) Does the improper integral $\int_1^\infty \frac{x}{x^{5/2}+1}\,dx$ converge or diverge? You do not need to compute the value if it converges. Provide some justification to your answer for full credit.

(+2 pts answer, +2 pts justification)

$\boxed{\text{Converges}}$

by limit comparison test

$$\frac{\frac{x}{x^{5/2}+1}}{1/x^{3/2}} = \frac{x^{5/2}}{x^{5/2}+1} \longrightarrow \underline{\underline{1}}.$$

6. Integrate. (8 pts. each)

(a) $\displaystyle\int \frac{x-1}{x^2+2x}\,dx$

partial fraction:

$$\frac{x-1}{x^2+2x} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)}$$

$A+B=1$. $A = -1/2$

$2A=-1$. $B = +3/2$

So integral equals $\displaystyle\int \frac{(-\frac{1}{2})}{x} + \frac{(3/2)}{x+2}\,dx$

$= \boxed{-\frac{1}{2}\ln(x) + \frac{3}{2}\ln(x+2) + C}$

(b) $\displaystyle\int \frac{x+1}{x^2+2x}\,dx$

u-sub

$\boxed{\begin{array}{l} u = x^2+2x \\ du = 2x+2 \end{array}}$

$= 2\displaystyle\int \frac{1}{u}\,du$

$= 2\ln|u| + C$

$= \boxed{2\ln|x^2+2x| + C}$