Learning objectives for MATH 1552

Course Learning Objectives

1. Students will master basic Calculus concepts, including integration techniques, convergence of integrals and infinite series, and Taylor's theorem.

2. Knowledge of the above concepts will be exhibited algebraically and geometrically.

3. Calculus concepts will be applied to solve physics, geometry, and numerical approximation problems.

4. Students will understand the usage of proper mathematical notation in relation to the above topics.

Unit Topics and Midterm Schedule

(I) For Midterm 1

- Sections 4.9, 5.1-5.6, 8.2-8.3. Riemann sums, the definite integral, integration by substitution, area between curves, integration by parts, integration of powers of trig functions, the Fundamental Theorem of Calculus.

(II) For Midterm 2

- Sections 8.4-8.5, 4.5, 8.8, 10.1-10.3. Trigonometric substitution, partial fractions, L'Hopital's Rule, improper integrals, sequences and infinite series, the integral test.

(III) For Midterm 3

- Sections 10.4-10.9. Comparison tests, ratio and root test, alternating series, power series, Taylor and MacLaurin Series.

(IV) Additional topics for Final Exam

- Sections 6.1-6.2, 7.2. Volumes of revolution by disk and shell method, separable Diff EQ.

Daily learning objectives

Week 1: Review of Derivatives/Anti-derivatives, Area Under the Curve, Sigma Notation

§4.8: Antiderivatives

1. Memorize the definition of antiderivative on page 281. **Definition:** A function F is an *antiderivative* of f on an interval I if $F'(x) = f(x)$ for all x in I.

2. Memorize and practice utilizing the formulas in Table 4.2 pg. 282 in §4.8, the antiderivative formulas.

TABLE 2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative	Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$	**8.** e^{kx}	$\frac{1}{k}e^{kx} + C$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$	**9.** $\frac{1}{x}$	$\ln \lvert x \rvert + C, \quad x \neq 0$
3. cos kx	$\frac{1}{k}\sin kx + C$	**10.** $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$	**11.** $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$	**12.** $\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, kx > 1$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$	**13.** a^{kx}	$\left(\frac{1}{k \ln a}\right)a^{kx} + C, a > 0, a \neq 1$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$		

3. Understand the reason for "$+C$" in the general antiderivative, accompanied by pictures of several functions who differ by a constant $f(x) + C$ and noticing that the slopes of all the functions are the same.

§5.1 and §5.2: Area Under the Curve Sigma Notation

1. Be able to compute left-endpoint, right-endpoint, upper-estimate, and lower-estimate finite sum approximations to areas under a curve $y = f(x)$ using n rectangles.

2. Be able to compute a midpoint approximation of the area under a curve $y = f(x)$ using n rectangles.

3. Practice using and recognizing sigma notation to express finite sums.

Week 2: The Definite Integral, The Fundamental Theorem of Calculus

§5.3: The Definite Integral

1. Understand how to use a sigma notation for a Riemann Sum in order to reconstruct the definite integral that is being approximated (similar to problems 1-8 on page 324 in §5.3).

2. Memorize the formulas for left-endpoint and right-endpoint approximations of definite integrals using Riemann Sums in Sigma notation.

3. Know and understand the sum, difference, constant multiple, and constant value rule for finite sums in Sigma notation.

4. Find a closed form for a finite sum using the Gauss formula $\sum_{i=1}^{n} k = \frac{n(n+1)}{2}$ and the formulas on pg. 312 for the first n squares and the first n cubes.

§5.4: The Fundamental Theorem of Calculus

1. Memorize the Fundamental Theorem of Calculus Part I & II, Theorem 4 on page 330-331.

THEOREM 4 (Continued)—The Fundamental Theorem of Calculus, Part 2 If f is continuous over $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

TH OREM 4—The Fundamental Theorem of Calcl us, Part 1 If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$:

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x). \qquad (2)$$

2. Be able to compute definite integrals with the FTC, using correct notation and clearly showing all steps.

3. Be able to compute $F'(x)$ given $F(x)$ expressed as a definite integral in the variable t with one of the limits as x or a function $f(x)$. The latter uses the chain rule, and the student should attempt to understand the proof of the general form of the FTC part I.

Week 3: Integration by Substitution, Area Between Curves

§5.5: Integration by Substitution

1. Set up the substitution "box" or other learning tool which has two parts u and du. For example in the box could be $u = 4x^2$ and $du = 8x\ dx$.

2. Gain experience choosing the u, and being able to find the du by differentiation, including the dx term for accurate substitutions.

3. Be able to see what can go wrong if the wrong u is chosen, how it is not possible to "pull out functions of x" from an integral if there are "extra terms".

§5.6: Area Between Curves

1. Be able to find intersection points of two or more curves by setting $y = y$ and then solving for x.

2. Set up and use a sign chart to determine which function is TOP and which is BOT.

3. Understand the meaning of the formula $A = \int_a^b \text{TOP} - \text{BOT}\ dx$, and how it is used to calculate area between the curves BOT $\leq$ TOP over the interval $[a, b]$.

Week 4: Integration by Parts, Integration of Products and Powers of Trig Functions

§8.2: Integration by Parts

1. Memorize the integration by parts formula

$$\int u\ dv = uv - \int v\ du$$

2. Memorize the ILATE acronym (Inverse trig, logs, algebra, trig, and exponential functions) and be able to recognize the parts of the integrand as each type.

3. Recognize an integral as a potential integration by parts integral by being able to perform mental math to quickly notice the du and v parts of the integration by parts technique.

4. Gain experience and intuition to realize when the u and dv have been chosen incorrectly because the integral is getting worse and not better after applying the IBP formula.

5. Be able to perform a "wrap around" integration by parts problem such as

$$\int \sin(x)e^x \, dx$$

where the integral appears after two iterations of integration by parts and then can be solved for.

6. Be able to compute a definite integral using the correct notation to evaluate uv in the IBP formula.

§8.3: Integration of Powers and Products of Trig Functions

1. Be able to recognize to use Pythagorus $\sin^2 x + \cos^2 x = 1$ to substitute and use u-substitution in integrals of the form $\int \sin^m(x) \cos^m(x) \, dx$, when one of m, n is odd.

2. Be able to use the *power reducing formulas*

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

and reduce to the previous case when both m, n are even in integrals of the form $\int \sin^m(x) \cos^m(x) \, dx$.

3. Be able to recognize problems of the form $\int \tan^m x \sec^n x \; dx$ as potentially being u-sub problems after substitutions using one of the modified Pythagorus formulas $\sec^2 x = 1 + \tan^2 x$ or $\tan^2 x = \sec^2 x - 1$.

4. Be able to perform any of the above integrations as a definite integral, implying routine understanding of trig function values given appropriately common inputs.

Week 5: Trig Sub, Midterm 1 (not including trig sub)

§8.4: Trigonometric Substitution

1. Be able to recognize and substitute integrals which are appropriate for a trigonometric substitution, based on the following table, which relies on the three formulas from Pythagorus: $\cos^2(x) = 1 - \sin^2(x)$, $\sec^2(x) = 1 + \tan^2(x)$, $\tan^2(x) = \sec^2(x) - 1$.

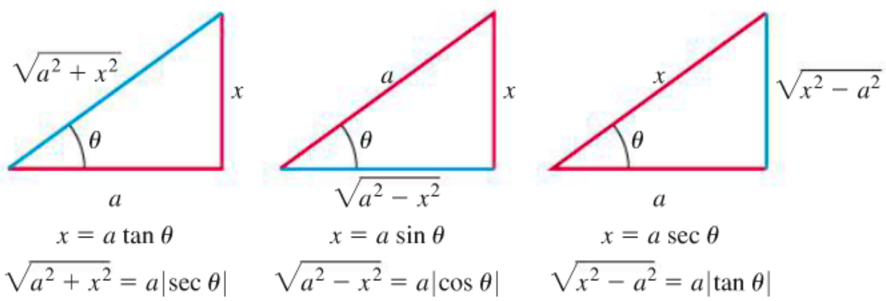

FIGURE 8.2 Reference triangles for the three basic substitutions identifying the sides labeled x and a for each substitution.

see	sub	don't forget
$a^2 - b^2x^2$	$x = \frac{a}{b}\sin\theta$	$dx = \frac{a}{b}\cos\theta\ d\theta$
$a^2 + b^2x^2$	$x = \frac{a}{b}\tan\theta$	$dx = \frac{a}{b}\sec^2\theta\ d\theta$
$b^2x^2 - a^2$	$x = \frac{a}{b}\sec\theta$	$dx = \frac{a}{b}\sec\theta\tan\theta\ d\theta$

2. After making the appropriate substitution for x AND dx, be able to simplify the resulting integral, and integrate to get the anti-derivative in terms of the variable θ.

3. Be able to use the reference triangle to substitute back into the variable x, giving the final answer in the same variable as the original question.

Week 5: Partial Fractions, L'Hopital's Rule

§8.5: Partial Fractions

1. Be able to set up the partial fraction decomposition for 2-3 non-repeated linear factors when the rational function is proper (degree of numerator<degree of denominator).

$$\frac{p(x)}{(x-c_1)(x-c_2)(x-c_3)} = \frac{A}{x-c_1} + \frac{B}{x-c_2} + \frac{C}{x-c_3}$$

2. Understand that solving for the constants A, B, C in above amounts to clearing denominators, collecting like terms and setting up and solving a system of linear equations.

3. Be able to set up and solve partial fraction decompositions for (proper) rational functions with repeated denominators, and irreducible quadratic terms, e.g.,

$$\frac{p(x)}{(x-c)^2(x^2+1)} = \frac{A}{x-c} + \frac{B}{(x-c)^2} + \frac{Cx+D}{x^2+1}.$$

4. Be able to set up, solve, and integrate for any combination of the above linear/quadratic and non/repeated denominators for proper rational functions, up to having to solve for about 4-5 constants.

5. Understand how to use polynomial long division to reduce an improper rational function into a polynomial plus a proper rational function.

§4.5: L'Hopital's Rule

1. Be able to recognize when L'Hopital's Rule can be applied, namely when the function $f(x)/g(x)$ is in an *indeterminant form* of $0/0$ or ∞/∞. Also, the student should see examples where applying L'Hop when the function is NOT indeterminant can lead to the incorrect answer!

2. Be able to (successively if need be) apply L'Hop Rule to evaluate limits,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

when f/g is an indeterminant form at $x = a$, for finite as well as infinite values of a, for both converging limits and limits which diverge to infinity.

Week 6: Improper Integrals, Sequences

§8.8: Improper Integrals

1. Be able to recognize and use the correct limit notation to evaluate improper integrals with an infinity in the limit of integration, e.g.,

$$\int_a^\infty f(x)\,dx = \lim_{N\to\infty} \int_a^N f(x)\,dx$$

2. Be able to evaluate improper integrals of the type above using L'Hop if necessary, by evaluating a definite integral with an arbitrary constant N as one of the limits of integration, and taking the limit as $N \to \infty$.

3. Be able to recognize an improper integral where $f(x)$ has a vertical asymptote in the interval $[a, b]$, and to set up and evaluate the improper integral using limits, as in e.g.,

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^3 \frac{1}{\sqrt{x}}\,dx.$$

4. Memorize and understand the derivation of the p-test, which states that for $a > 0$,

$$\int_a^\infty \frac{1}{x^p}\,dx = \begin{cases} +\infty\text{DNE} & \text{if } 0 < p \le 1, \\ \text{converges} & \text{if } p > 1. \end{cases}$$

§10.1: Sequences

1. Be able to find the closed form of a geometric or arithmetic sequence, including the starting index value $n = n_0$ of the sequence terms.

$$\begin{aligned} a_n &= a \cdot r^n && \text{(geometric)} \\ a_n &= m \cdot n + b && \text{(arithmetic)} \end{aligned}$$

2. Be able to recognize geometric and arithmetic sequences by the fact that the terms of a geometric sequence have a common ratio $a_{n+1}/a_n \equiv r$ and the terms of an arithmetic sequence have a common difference $a_{n+1} - a_n \equiv m$.

3. Practice recognizing harder sequences such as the following:

$$\begin{array}{rcl} \text{(one off from a square)} & 3, 8, 15, 24, 35, 48, 63, \ldots & a_n = n^2 - 1,\ n \geq 2, \\ \text{(alternating 0's and 1's)} & 0, 1, 0, 1, 0, 1, \ldots & a_n = \dfrac{1 + (-1)^n}{2},\ n \geq 0, \\ \text{(odd and even numbers)} & \dfrac{1}{2}, \dfrac{3}{4}, \dfrac{5}{6}, \dfrac{7}{8}, \ldots & a_n = \dfrac{2n - 1}{2n},\ n \geq 1, \\ \text{(factorials)} & \dfrac{1}{1}, \dfrac{1}{2}, \dfrac{1}{6}, \dfrac{1}{24}, \dfrac{1}{120}, \ldots & a_n = \dfrac{1}{n!},\ n \geq 0. \end{array}$$

Week 7: Infinite Series, the Integral Test

§10.2: Infinite Series

1. Be able to convert a given series written out in expanded form using "+" plus signs into a series written in a more compact form using Σ-notation, by recognizing the terms of the series as a sequence and finding the closed form of this sequence (including the start-value $n = n_0$).

2. Memoize and become familiar with the usage of the geometric series formula,

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1 - x},\ |x| < 1,$$

as well as it's variants involving: different starting index, common factor on each term, which can be summarized in the formula

$$\sum_{n=n_0}^{\infty} ax^n = \frac{ax^{n_0}}{1 - x},\ |x| < 1.$$

3. Understand the derivation of the geometric series formula, e.g., via

$$\begin{aligned} (1 - x)(1 + x + x^2 + \cdots + x^N) &= 1 - x^{N+1}, \\ \implies \sum_{n=0}^{N} x^n &= \frac{1 - x^{N+1}}{1 - x}, \\ \implies \sum_{n=0}^{\infty} x^n &= \frac{1}{1 - x},\ |x| < 1. \end{aligned}$$

Specifically, students should become familiar with several cases where the geometric series $\sum r^n$ diverges, especially when $|r| = 1$.

4. Understand that some infinite sequences converge and some diverge, using simple examples that can be explained heuristically, such as

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2,$$
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \cdots \qquad \text{diverges}^*$$

The second series can be seen to diverge since

$$\begin{aligned} &1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} \cdots \\ \geq &1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \cdots \\ = &1 + \frac{1}{2} \qquad + \frac{1}{2} \qquad\qquad + \frac{1}{2} + \cdots . \end{aligned}$$

5. Memorize the divergence test (a.k.a., the n-th term test), which states that

$$\begin{array}{l} \text{If } \sum a_n \text{ converges,} \\ \text{then } a_n \to 0, \end{array} \iff \begin{array}{l} \text{If } a_n \not\to 0, \\ \text{then } \sum a_n \text{ diverges.} \end{array}$$

6. Understand that the converse of the above statement is FALSE. Namely, the statement "If $a_n \to 0$, then $\sum a_n$ converges" is FALSE. Also, students should be able to quickly come up with several examples which illustrate the falsity of this statement, e.g., $\sum \frac{1}{n}$ is a diverging series even though $a_n = \frac{1}{n} \to 0$.

7. Be able to write out the first several terms of a series written in Σ-notation, in expanded form with "+" plus signs.

§10.3: The Integral Test

1. Understand the integral test for series as meaning that in order to evaluate whether a given series $\sum a_n$ converges or diverges, one can instead evaluate the improper integral $\int_{n_0}^{\infty} f(x)\ dx$ whose integrand $f(x)$ is the function which obtains the terms of the series $f(n) = a_n$.

2. Understand the application of the p-test with the integral test yields the logical equivalence between the two statements below,

$$\int_a^\infty \frac{1}{x^p}\,dx = \begin{cases} +\infty \text{ DNE} & \text{if } 0 < p \leq 1, \\ \text{converges} & \text{if } p > 1. \end{cases}$$

$$\sum_{n=1}^\infty \frac{1}{n^p}\,dx = \begin{cases} +\infty \text{ DNE} & \text{if } 0 < p \leq 1, \\ \text{converges} & \text{if } p > 1. \end{cases}$$

3. Be able to apply the integral test to determine whether a given series converges or diverges, for appropriately chosen examples, using u-sub, IBP or other integration techniques to evaluate the improper integral.

4. Have familiarity with the heuristic proof of the integral test, involving drawing a left/right-endpoint Riemann sum (upper/lower) with arbitrarily many rectangles of base-width $\Delta x = 1$, so that the area of the Riemann sum is exactly the value of the series, and the area under the curve is the value of the improper integral. Using an upper vs. lower RS shows that the improper integral is either less vs. greater than the RS value, depending on whether you want to show convergence vs. divergence of the series (which is the RS).

Week 8: Comparison Tests, Midterm 2 (not including comparison tests)

§10.4: Comparison Tests

1. Memorize the statement, be able to use, and understand the heuristic proof of the direct comparison test, which is the statement regarding determining whether a given series $\sum a_n$ converges or diverges below.

$$\text{If } a_n \leq b_n \text{ and } \sum b_n \text{ converges, then } \sum a_n \text{ also converges,}$$
$$\text{and if } a_n \geq b_n \text{ and } \sum b_n \text{ diverges, then } \sum a_n \text{ also diverges.}$$

2. Memorize the statement and be able to use in practice the limit comparison test, which is the statement regarding determining whether a given series

$\sum a_n$ converges or diverges below.

$$\text{If } \lim_{n\to\infty} \frac{a_n}{b_n} = L, \text{ and } 0 < L < \infty,$$

$$\text{then } \sum a_n \text{ and } \sum b_n \text{ either both converge, or both diverge.}$$

3. Practice recognizing which series are properly suited to use the direct comparison test, based on recognizing whether the given series should converge or diverge and the Mnemonic devices "top bigger whole fraction bigger" and "bottom smaller whole fraction bigger", etc.

Week 9: Ratio and Root Tests, Alternating Series

§10.5: Ratio and Root Tests

1. Memorize the statement and be able to use in practice the Ratio Test, which is the statement regarding determining whether a given series $\sum a_n$ converges or diverges below.

$$\text{Suppose that } \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho,$$

$$\text{then the series } \sum a_n \begin{cases} \text{CONVERGES} & \text{if } \rho < 1, \\ \text{test inconclusive} & \text{if } \rho = 1, \\ \text{DIVERGES} & \text{if } \rho > 1. \end{cases}$$

2. Memorize the statement and be able to use in practice the Root Test, which is the statement regarding determining whether a given series $\sum a_n$ converges or diverges below.

$$\text{Suppose that } \lim_{n\to\infty} \sqrt[n]{|a_n|} = \rho,$$

$$\text{then the series } \sum a_n \begin{cases} \text{CONVERGES} & \text{if } \rho < 1, \\ \text{test inconclusive} & \text{if } \rho = 1, \\ \text{DIVERGES} & \text{if } \rho > 1. \end{cases}$$

3. Practice recognizing when the ratio/root test should be used to determine whether a given series converges or diverges. In particular, the ratio test should be used if the terms a_n involve factorials, and the root test should be used if the terms a_n involve expressions such as n^n where the index n shows up as (part of) the base *and* exponent of a term.

§10.6: Alternating Series

1. Learn to apply and recognize applicable series for the alternating series test: the alternating series $\sum(-1)^n a_n$ converges if $a_n \geq 0$ (non-negative), $a_{n+1} < a_n$ (decreasing), and $\lim a_n = 0$.

2. Be able to distinguish conditional/absolute convergence of an alternating series. Namely, an alternating series $\sum(-1)^n a_n$ converges absolutely if $\sum a_n$ converges, and converges conditionally if $\sum(-1)^n a_n$ converges but $\sum a_n$ diverges.

3. Become familiar with examples of conditionally convergent series, so that the student can quickly produce examples of series which converge conditionally or absolutely.

4. Memorize the alternating series error approximation formula. Namely, if $L = \sum_{n=0}^{\infty}(-1)^n a_n$ is the value of an alternating series, with a_n satisfying the conditions of the alternating series test, and $S_N = \sum_{n=0}^{\infty}(-1)^n a_n$ the partial sum of the series, then

$$|L - S_N| \leq a_{N+1}.$$

Week 10: Spring Break (no class)

Week 11: Power Series, Taylor Series

§10.7: Power Series

1. Be able to compute the radius and interval of convergence of a given power series $\sum a_n(x-a)^n$, typically by using the ratio test.

2. Understand that in order to determine the interval of convergence, one must test the endpoints of the interval of convergence $(a-R, a+R)$ given by the ratio test, since at the endpoints of this interval the ratio test is inconclusive.

3. Gain intuition through examples about the meaning of radius and interval of convergence. Namely, that $f(x) = \sum a_n(x-a)^n$ is a *function of the independent variable* x, and so the interval of convergence is the range of x-values for which this function is defined (*i.e.*, those input values for which there is an output value), and that the radius of convergence specifies the distance from the center $x = a$ that still obtains a finite output. Specifically, students should see through examples (using either software, spreadsheets, or online tools) that the closer the x-value input x_0 is to the center, the less terms you need in the series to get a good approximation for the final output value of the series $f(x_0)$.

§10.8-10.9: Taylor Series (one day)

1. Memorize the definition of the Taylor series expansion at $x = 0$,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n,$$

as well as the Taylor series expansion at $x = a$,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n.$$

2. Memorize and become familiar with the derivation of familiar Taylor series, namely

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{1}{n!} x^n, \\ \frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n, \\ \sin(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \quad &\text{(if time permits)} \\ \cos(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}. \quad &\text{(if time permits)} \end{aligned}$$

3. Be able to recognize or readily compute the interval of convergence of each of the above series.

Week 12: Taylor Series (cont.)

§10.8-10.9: Taylor Series (all week)

1. Practice using the Taylor series formula to find the Taylor series for various functions; for example, the functions $\sin(x), \cos(x)$ centered at $x = 0$ and

$\ln(x)$ centered at $x = 1$ or $x = 2$.

2. Practice using known Taylor series to find other representations, for example, for $x^2 \sin(x^4)$ and $\frac{x}{4+x^2}$, by using algebraic manipulations and substitution into known formulas.

3. Understand how to take a derivative and integrate a Taylor series term-wise, in order to obtain representations for various functions, such as $\tan^{-1}(x)$ and $\frac{1}{(1-x)^2}$, and $\ln(x)$.

4. Be able to find a Taylor polynomial of degree N, especially for functions which may not have an easy to find Taylor series, such as $\sqrt{x}$.

5. Compute an upper bound on the error of a degree N Taylor polynomial at a point $x = x_0$, using the Remainder Theorem

$$|R_N(x)| \leq \max |f^{(N+1)}(c)| \frac{|x_0 - a|}{(N+1)!},$$

as well as the alternating series error estimate $|L - S_N| \leq a_{N+1}$.

6. Be able to find the smallest degree N that a Taylor polynomial must have for a given function in order for the approximation to be within a certain threshold.

Week 13: Volumes of Revolution, Midterm 3 (not including Volumes of Revolution)

§6.1: Volumes of Revolution (one day)

1. Practice sketching simple polynomial and trigonometric functions in order to visualize regions revolved about horizontal or vertical axes of rotation.

2. Be able to compute simple volumes of revolution for regions revolved about the x-axis or y-axis, using the disk and washer methods.

Week 14: Volumes of Revolution (cont.) and Separable Differential Equations

§6.1: Volumes of Revolution (one day)

1. Be able to compute more complicated volumes of revolution for regions revolved about any vertical or horizontal axis, using either disk, washer, or the cylindrical shell method.

§7.2: Separable Differential Equations

1. Gain experience separating variables for several types of polynomial or exponential equations, in order to set up solving a separable differential equation.

2. Solve separable differential equations and initial value problems by integrating both sides of a separable differential equation and solving for the dependent variable.

3. Understand that the constant of integration "$+C$" can sometimes be buried inside the function, $y = f(x)$; also be able to recognize and avoid errors occurring from solving for C in one step of the work but plugging in this C-value in the final expression.