1. Integrate using any method from class. (16 pts. each)

(a) $\int 5x\sec^2(3x)\,dx$ $= \frac{5x}{3}\tan(3x) - \int \frac{5}{3}\tan(3x)\,dx$

IBP Box

$u = 5x \qquad dv = \sec^2(3x)\,dx$

$du = 5\,dx \qquad v = \frac{1}{3}\tan(3x)$

$$= \boxed{\frac{5x}{3}\tan(3x) - \frac{5}{9}\ln(\sec(3x)) + C}$$

(b) $\int \frac{x^2}{(x^2+9)^{3/2}}\,dx$ $= \int \frac{9\tan^2\theta}{(9\tan^2\theta+9)^{3/2}} \cdot 3\sec^2\theta\,d\theta$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$$= \int \frac{27\tan^2\theta\sec^2\theta}{(9\sec^2\theta)^{3/2}}\,d\theta = \int \frac{\cancel{27}\tan^2\theta\,\cancel{\sec^2\theta}}{\cancel{27}\sec^{\cancel{3}}\theta}\,d\theta$$

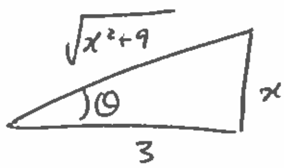
$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta = \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta = \int\left(\sec\theta - \frac{1}{\sec\theta}\right)d\theta$$

$$= \int \sec\theta - \cos\theta\,d\theta$$

$\tan\theta = x/3$

$$= \ln(\sec\theta + \tan\theta) - \sin\theta + C$$

$$= \boxed{\ln\left(\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right) - \frac{x}{\sqrt{x^2+9}} + C}$$



2. Integrate using any method from class. (16 pts. each)

(a) $\int \frac{3x+8}{x^3+x}\,dx = \int \frac{3x+8}{x(x^2+1)}\,dx = \int \frac{Ax+B}{x^2+1} + \frac{C}{x}\,dx$

$(Ax+B)x + C(x^2+1) = 3x+8$

$(A+C)x^2 + Bx + C = 3x+8$

$$\left.\begin{array}{l} A+C=0 \\ B=3 \\ C=8 \end{array}\right\} \Rightarrow \begin{array}{l} A=-8 \\ B=3 \\ C=8 \end{array}$$

$= \int \left(\frac{-8x+3}{x^2+1} + \frac{8}{x} \right) dx$

$= \int \left(\frac{-8x}{x^2+1} + \frac{3}{x^2+1} + \frac{8}{x} \right) dx$

$= \boxed{-4\ln(x^2+1) + 3\tan^{-1}(x) + 8\ln|x| + C}$

(b) $\int \sin^4(x)\cos^3(x)\,dx = \int \sin^4(x)\cos^2(x)\cdot\cos(x)\,dx$

$= \int \sin^4 x\,(1-\sin^2 x)\cos x\,dx = \int u^4(1-u^2)\,du$

$= \int u^4 - u^6\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + C$

$= \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$

3. Either evaluate the integral or show that it diverges. (16 pts.)

$$\int_1^\infty \frac{1-\ln(x^2)}{x^2}\,dx$$

$$= \int_1^\infty \frac{1-2\ln x}{x^2}\,dx = \int_1^\infty \frac{1}{x^2} - 2\int_1^\infty \frac{\ln x}{x^2}\,dx$$

↑ IBP Box

$u = \ln x \quad dv = \frac{1}{x^2}$

$du = \frac{1}{x}\,dx \quad v = \frac{-1}{x}$

$$= \frac{-1}{x}\Big|_1^\infty - 2\left(\frac{-1}{x}\ln x\Big|_1^\infty - \int_1^\infty \frac{-1}{x}\cdot\frac{1}{x}\,dx\right)$$

$$= (0-(-1)) - 2\left((0-0) - \left(\frac{-1}{x}\Big|_1^\infty\right)\right)$$

$$= 1 - 2(0-(0-1))$$

$$= 1 - 2 = \boxed{-1}$$

Comparison test ok to CHECK but still need to evaluate!

$$(\star)\ \int_1^\infty \frac{1-\ln(x^2)}{x^2}\,dx \le \int_1^\infty \frac{1}{x^2}\,dx \quad \text{b/c } 1-\ln x^2 \le 1$$

$$= \frac{-1}{x}\Big|_1^\infty = 0-(-1) = 1$$

So converges

OR conv by p-t

Conclude that $(\star)$ converges by direct comparison.

4. Evaluate the limit.

$$\lim_{x\to 0} \frac{x\tan^{-1}(x)}{x^2 - \cos x} \quad \boxed{= \ 0}$$

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5. For what values of p does the integral $\int_6^\infty \frac{1}{x(\ln x)^p}\,dx$ converge? (10 pts.)

$$= \int_{\ln 6}^{\infty} \frac{1}{u^p}\,du \qquad \text{Converges if } \boxed{p > 1}$$

Bonus: (5 pts.) Evaluate $\int \sec^3(x)\,dx$.

IBP box

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x \qquad v = \tan x$

WRAP AROUND

$\star = u\text{———}u - \star$

$2\star = u\text{———}u$

$$= \tan x \sec x - \int \tan x \cdot \sec x \tan x\,dx$$

$$= \tan x \sec x - \int \tan^2 x \sec x\,dx$$

$$= \tan x \sec x - \int (\sec^2 x - 1)\sec x\,dx$$

$$= \tan x \sec x - \int \sec^3 x\,dx + \int \sec x\,d$$

$$= \tan x \sec x - \ln(\sec x + \tan x) - \int \sec^3 x$$

So $\boxed{\int \sec^3 x\,dx = \frac{1}{2}\tan x \sec x + \frac{1}{2}\ln(\sec x + \tan x) + C}$