1. Find the value of the series. (16 pts.)

$$\sum_{n=2}^{\infty} \frac{2^{2n-2}}{5 \cdot 10^{n-2}}$$

$$= \frac{100}{5 \cdot 4} \sum_{n=2}^{\infty} \left(\frac{4}{10}\right)^n$$

$$= 5 \sum_{n=2}^{\infty} \left(\frac{2}{5}\right)^n$$

$$= 5 \left(\frac{1}{1 - 2/5} - 1 - 2/5 \right)$$

$$= 5 \left(\frac{5}{3} - 1 - \frac{2}{5} \right)$$

$$= \cancel{5} \left(\frac{25 - 15 - 6}{\cancel{15}\ 3} \right)$$

$$= \cancel{5}\ \cancel{4}\ \boxed{4/3}$$

2. Do the sequences $\{a_n\}$ with $n \geq 0$ converge or diverge? If they converge, find the limit. Justify your answers. (7 pts. each)

(a) $a_n = \dfrac{(-1)^n}{\sqrt{n}+1}$

$|a_n| \to 0$ so $\boxed{a_n \to 0.}$

(b) $a_n = \dfrac{n^2}{(2n+1)(2n+2)}$

$\boxed{a_n \to \frac{1}{4}}$ by L'Hopital's rule for example.

3. Determine if the given series converge or diverge. Fully justify your answer using any of the convergence tests from class in order to receive full credit. (12 pts. each)

(a) $\sum_{n=2}^{\infty} \dfrac{n}{\sqrt{n^3 - 1}}$ $\geq \sum_{n=2}^{\infty} \dfrac{n}{\sqrt{n^3}} = \sum_{n=2}^{\infty} \dfrac{1}{n^{1/2}}$ diverges by p-test.

Guess: $\dfrac{n}{n^{3/2}} = \dfrac{1}{n^{1/2}}$ should get divergence.

So $\boxed{\text{diverges}}$ by direct comparison and p-test w/ $\underline{p = 1/2}$.

(b) $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^{3/2}}$

$\boxed{\text{Converges}}$

integral test $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^{3/2}}\,dx = \int_{\ln 2}^\infty \frac{1}{u^{3/2}}\,du$

converges by p-test w/ $p = 3/2$.

4. Determine if the given alternating series converges absolutely, conditionally, or diverges. Fully justify your answer using any of the convergence tests from class in order to receive full credit. (15 pts.)

$$\sum_{n=2}^{\infty} \frac{(-1)^n\, n^2}{n^3+1}$$

$a_n = \frac{n^2}{n^3+1} \longrightarrow 0$ so alternating series test $\Longrightarrow$ converges.

but $\sum_{n=2}^{\infty} \frac{n^2}{n^3+1}$ diverges by integral test $\int_2^{\infty} \frac{n^2}{n^3+1}\,dx = \frac{1}{3}\int_9^{\infty} \frac{1}{u}\,du$

diverges by p-test w/ $p = 1$.

So the series $\boxed{\text{converges conditionally}}$

5. Find the interval and radius of convergence of the power series. (15 pts.)

$$\sum_{n=2}^{\infty} \left(\frac{n}{n+1}\right) \frac{(x+3)^n}{4^n}$$

ratio test

$$\frac{a_{n+1}}{a_n} = \left(\frac{n+1}{n+2}\right) \cdot \frac{(x+3)^{\cancel{n+1}}}{4^{\cancel{n+1}}} * \left(\frac{n+1}{n}\right) \frac{\cancel{4^n}}{\cancel{(x+3)^n}}$$

$$= \frac{(n+1)^2}{n(n+2)} \cdot \frac{x+3}{4} \longrightarrow \frac{x+3}{4} = L$$

$|L| < 1$ need $\left|\frac{x+3}{4}\right| < 1 \Rightarrow |x+3| < 4$

$x \in (-7, 1)$

Check endpoints $x = +1$ $\sum_{n=2}^{\infty} \frac{n}{n+1}$ diverges

$x = -7$ $\sum_{n=2}^{\infty} \left(\frac{n}{n+1}\right)(-1)^n$ also diverges

$\boxed{x \in (-7, 1) \quad \text{radius } R = 4}$

6. For which value of p does the following series converge to 16. (16 pts.)

$$\sum_{n=0}^{\infty} \frac{14}{4^{pn}} = 14 \sum_{n=0}^{\infty} \left(\frac{1}{4^p}\right)^n$$

$$= 14 \cdot \left(\frac{1}{1-(1/4^p)}\right) \overset{\text{need}}{=} 16$$

$$\boxed{p = 3/2}$$

So $\dfrac{1}{1-(1/4^p)} = \dfrac{16}{14} = \dfrac{8}{7}$

$\dfrac{7}{8} = 1 - \left(\dfrac{1}{4^p}\right)$

$\dfrac{1}{4^p} = 1 - \dfrac{7}{8} = \dfrac{1}{8}$

$4^p = 8$

$\log_4(4^p) = \log_4 8$

$p = \log_4 8 = 3/2$

Since $4^{3/2} = 8$.

Bonus (5 pts.): How many terms must be used to determine the sum of the entire series with an error of less than 0.08?

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$$

$n = 6$ first $|a_n| < 0.08$

So $\sum_{n=1}^{6-1} (-1)^n \frac{2^n}{n!}$ is within 0.08 of the sum.

$6 - 1 = \boxed{5}$

Need $a_n = \dfrac{2^n}{n!} < 0.08$

$n = 5$?

$\dfrac{32}{120} \approx \dfrac{1}{4}$ too big

n	2^n	n!
1	2	1
2	4	2
3	8	6
4	16	24
5	32	120
6	64	720
7	128	5040

$n = 6$

$\dfrac{64}{720} = \dfrac{32}{360} = \dfrac{16}{180} = \dfrac{8}{90} < \dfrac{8}{100} = 0.08$ ✓