Math 1552 **Exam 1** Summer-SOUP 2018

Instructor: Sal Barone

Name: Key

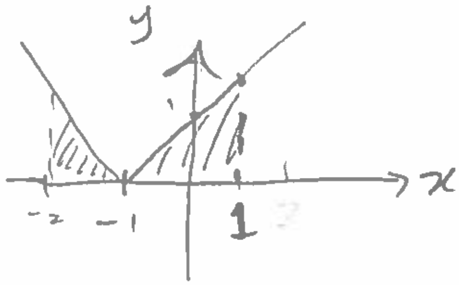
GT username: ____________________

1. No books or notes are allowed.
2. No electronic devices are allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Please $\boxed{\text{BOX}}$ your answers.
5. The exam consists of 105 points but your score will be out of 100, there is a 5 pt. bonus question at the end.
6. Good luck!

Page	Max. Possible	Points
1	22	
2	36	
3	17	
4	30	
Total	105	

1. Evaluate. (*Hint: graph the function.*) (10 pts.)

$$\int_{-2}^{1} |x+1|\, dx$$



$$= \int_{-2}^{-1} -(x+1)\, dx + \int_{-1}^{1} (x+1)\, dx$$

$$= -\tfrac{1}{2}x^2 - x \Big|_{-2}^{-1} + \tfrac{1}{2}x^2 + x \Big|_{-1}^{1}$$

$$= \left[\left(-\tfrac{1}{2}+1\right) - \cancel{(-2+2)}\right] + \left[\left(\tfrac{1}{2}+1\right) - \left(\tfrac{1}{2}-1\right)\right]$$

$$= \boxed{5/2}$$

alt soln. (geometrically)

first area $\frac{1}{2}(1)^2$

second area $\frac{1}{2}(2)^2$

add them up $\frac{1}{2} + \frac{4}{2} = \boxed{5/2}$

2. Find $F'(3)$ where (12 pts.)

$$F(x) = \int_{\pi/6}^{4x} \cos^2\left(\frac{\pi t}{6}\right)\, dt.$$

$$F'(x) = \cos^2\left(\frac{\pi \cdot 4x}{6}\right) * 4$$

$$= 4\cos^2\left(\frac{2\pi \cdot x}{3}\right)$$

So

$$F'(3) = 4\cos^2\left(\frac{2\pi \cdot \cancel{3}}{\cancel{3}}\right) = \boxed{4}$$

3. Evaluate the indefinite integrals. (12 pts. each)

(a) $\int \frac{4r^2}{\sqrt{1-r^3}}\,dr \;=\; -\frac{4}{3}\int \frac{1}{\sqrt{u}}\,du = -\frac{4}{3}\cdot\int u^{-1/2}\,du$

$u = 1-r^3$

$du = -3r^2\,dr$

$-\frac{1}{3}\,du = r^2\,dr$

$= -\frac{4}{3}2u^{1/2} + C$

$= \boxed{-\frac{8}{3}(1-r^3)^{1/2} + C}$

(b) $\int \sec^2(2\theta)\tan(2\theta)\,d\theta \;=\; \int \sec(2\theta)\cdot\sec(2\theta)\tan(2\theta)\,d\theta$

$u = \sec(2\theta)$

$du = 2\cdot\sec(2\theta)\tan(2\theta)\,d\theta$

$= \int u\cdot\frac{1}{2}\,du = \frac{1}{4}u^2 + C$

$= \boxed{\frac{1}{4}\sec^2(2\theta) + C.}$

alt. use $u = \tan(2\theta)$

$du = 2\tan(2\theta)$

$= \int u\cdot\frac{1}{2}\,du = \boxed{\frac{1}{4}\tan^2(2\theta) + C}$

also correct (use identity to verify

(c) $\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx$

$= \int u^{1/3}\cdot 2\,du$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$= 2\cdot\frac{3}{4}u^{4/3} + C$

$= \boxed{\frac{3}{2}(1+\sqrt{x})^{4/3} + C}$

4. (a) Find a closed formula for the approximate area under $y = x(x-1)$ over the interval $[0,1]$ using n rectangles and a right-endpoint evaluation Riemann Sum.

(8 pts.)

$y = x^2 - x$

$R_n = \sum_{i=1}^{n} \Delta x\, f(x_i)$

$\Delta x = \frac{b-a}{n} = \frac{1}{n}$

$x_i = a + i * \Delta x = i/n$

$$R_n = \sum_{i=1}^{n} \frac{1}{n}\left[\left(i/n\right)^2 - \left(i/n\right)\right]$$

$$= \frac{1}{n^3}\sum_{i=1}^{n} i^2 - \frac{1}{n^2}\sum_{i=1}^{n} i$$

$$= \frac{1}{n^{\cancel{3}2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6} - \frac{1}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)}{2}$$

$$= \boxed{\frac{(n+1)(2n+1)}{6} - \frac{n+1}{2n}}$$

(b) Find the exact area from above by taking a limit in part *(a)*.

(4 pts.)

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[\frac{(n+1)(2n+1)}{6} - \frac{n+1}{2n}\right]$$

$$= \frac{2}{6} - \frac{1}{2} = \boxed{\frac{-1}{6}}$$

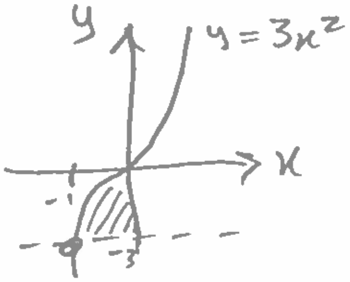
(c) Find the exact area from above again (check your answer) by using the Fundamental Theorem of Calculus.

(5 pts.)

$$\int_0^1 (x^2 - x)\, dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{2} = \boxed{-1/6}$$

5. Find the area bounded by $y = 3x^3$, the line $y = -3$ and the y-axis. (15 pts.)



intersection point when

$$-3 = 3x^3 \Rightarrow x^3 = -1$$

$$\Rightarrow \boxed{\begin{array}{l} x = -1 \\ y = -3 \end{array}}$$

$$\int_{-1}^{0} \left(3x^3 - (-3)\right)\, dx = \int_{-1}^{0} (3x^3 + 3)\, dx$$

$$= \frac{3x^4}{4} + 3x \Big|_{-1}^{0} = 0 - \left(\frac{3}{4} - 3\right) = \boxed{9/4}$$

6. Find the average value of $y = \frac{\ln(4x)}{x}$ over the interval $[1, e^2]$. (10 pts.)

$$\frac{1}{e^2-1}\int_1^{e^2} \frac{\ln(4x)}{x}\, dx = \frac{1}{e^2-1}\int_*^* u\, du = \frac{1}{e^2-1}\cdot\frac{1}{2}\cdot u^2 \Big|_*^*$$

$u = \ln(4x)$

$du = \frac{1}{x}\, dx$

$$= \frac{1}{e^2-1}\cdot\frac{1}{2}\cdot\left(\ln(4x)\right)^2 \Big|_1^{e^2}$$

$$= \frac{1}{2(e^2-1)}\left[\left(\ln(4e^2)\right)^2 - \left(\ln(4)\right)^2\right]$$

$$= \frac{1}{2(e^2-1)}\left[\left(\ln 4 + \ln(e^2)\right)^2 - (\ln 4)^2\right] = \frac{1}{2(e^2-1)}\left[4\ln(4) + \underbrace{\left(\ln(e^2)\right)^2}_{(2)^2}\right] = \boxed{\frac{2\ln 4 + 2}{e^2-1}}$$

Bonus: (5 pts.) Express the following limit as a definite integral but do not evaluate.

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{3}{n}\left(4\log\left(2 + \frac{3i}{n}\right)\right)$$

$$\boxed{\int_2^5 4\log(x)\, dx}$$