Quiz 3

1. Integrate. (8 pts.)

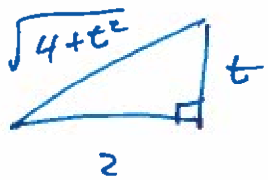
$$\int \frac{1}{\sqrt{t^2+4}}\,dt$$

$t = 2\tan\theta$

$dt = 2\sec^2\theta \, d\theta$

$$= \int \frac{1}{\sqrt{4\tan^2\theta + 4}} \cdot 2\sec^2\theta \, d\theta$$

$\tan\theta = t/2$



$$= \int \sec\theta \, d\theta = \int \sec\theta \cdot \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\, d\theta = \int \frac{1}{u}\, du$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$\boxed{= \ln\left|\frac{\sqrt{4+t^2}}{2} + \frac{t}{2}\right| + C}$$

2. Find the value. Simplify your answer. (7 pts.)

$$\int_{1/2}^{1} \frac{x+4}{x^2+x}\,dx$$

$$\frac{x+4}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$x+4 = A(x+1) + Bx$$
$$= (A+B)x + A$$

$A = 4 \quad B = -3$

$$= \int_{1/2}^{1} \left(\frac{4}{x} - \frac{3}{x+1}\right) dx$$

$$= 4\ln(x) - 3\ln(x+1)\Big|_{1/2}^{1}$$

$$= 4\cancel{\ln(1)}^{\,0} - 3\ln(2) - \left(4\ln(1/2) - 3\ln(3/2\right.$$

$$= -3\ln(2) + 4\ln(2) + 3\ln(3) - 3\ln(2)$$

$$= \boxed{-2\ln(2) + 3\ln(3)}$$

3. Use the formula

$$|E_T| \leq \frac{M(b-a)^3}{12n^2},$$

where $M \geq |f''|$ on $[a, b]$, to find the minimum number of trapezoids required to estimate

$$\int_{-2}^{1} (t^3 + t)\, dt$$

to within an error of 10^{-4}. (5 pts.)

$$|E_T| \leq \frac{M(1-(-2))^3}{12 \cdot n^2} = \frac{M \cdot 3^3}{12 \cdot n^2} = \frac{\cancel{12} \cdot 27}{\cancel{12} \cdot n^2} \leq 10^{-4}$$

$f(t) = t^3 + t$

$f'(t) = 3t^2 + 1$

$f''(t) = 6t$

$|f''(t)| \leq 12$ on $[-2, 1]$.

$\underline{\underline{M = 12}}$. (not 6)

Solve:

$$\frac{27}{n^2} \leq 10^{-4}$$

$$n^2 \geq 27 \cdot 10^4$$

$$\boxed{n \geq \sqrt{27} \cdot 100 \geq 5}$$