Quiz 4

1. **(a)** Write the series in sigma notation, then **(b)** find the value of the series. (8 pts.)

$$\frac{3}{5}+\frac{3}{25}+\frac{3}{125}+\frac{3}{625}+\dots$$

a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{5^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{5^n} = \frac{3}{5}\sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n = \frac{3}{5}\left(\frac{1}{1-1/5}\right) = \frac{3}{5}\left(\frac{1}{4/5}\right) = \frac{3}{\cancel{5}}\cdot\frac{\cancel{5}}{4} = \boxed{\frac{3}{4}}$

OR $\displaystyle = 3\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n = 3\left(\frac{1}{1-1/5}-1\right) = 3\left(\frac{5}{4}-1\right) = 3\left(\frac{1}{4}\right) = \boxed{\frac{3}{4}}$

2. Determine if the given series converges or diverges. Fully justify your work and provide *(a)* the test you are using, *(b)* a supporting sentence, and *(c)* show complete work. (6 pts.)

$$\sum_{n=2}^{\infty}\frac{e^n}{1+e^{2n}}$$

(NOTE: Limit comparison w/ $b_n = 1/e^n$ also works)

Integral test

$u = e^x$, $du = e^x\,dx$

$$\int_2^{\infty}\frac{e^x}{1+e^{2x}}\,dx = \int_{e^2}^{\infty}\frac{1}{1+u^2}\,du = \tan^{-1}(u)\Big|_{e^2}^{\infty}$$

$= \pi/2 - \tan^{-1}(e^2)$ converges.

By the integral test, since the integral converges the series also converges.

#1

Direct comparison

$$a_n = \frac{e^n}{1+e^{2n}} \le \frac{e^n}{e^{2n}} = \frac{1}{e^n} = b_n$$

$\sum b_n = \sum \frac{1}{e^n} = \sum\left(\frac{1}{e}\right)^n$ converges by geo. series test w/ $r = 1/e < 1$.

By direct comparison w/ $b_n = 1/e^n$, since $a_n \le b_n$ and the series $\sum b_n$ converges by geo. series test w/ $r = 1/e < 1$, the

3. Determine if the given series converges or diverges. Fully justify your work and provide *(a)* the test you are using, *(b)* a supporting sentence, and *(c)* show complete work. (6 pts.)

$$\sum_{n=3}^{\infty} \frac{1}{\sqrt{n+1}\sqrt{n-1}}$$

Direct comparison:

$$a_n = \frac{1}{\sqrt{n+1}\cdot\sqrt{n-1}} = \frac{1}{\sqrt{n^2-1}} \geq \frac{1}{\sqrt{n^2}} = \frac{1}{n} = b_n.$$

Since $\sum b_n = \sum \frac{1}{n}$ is the harmonic series, it diverges (p-test w/ $p=1$).

Therefore, the series $\sum a_n$ diverges by the direct comparison test, with $b_n = 1/n$ the harmonic series and $a_n \geq b_n$.