Quiz 6

1. Find the Taylor series at $x = 0$ (aka, the MacLaurin series) for the function below. Be clear in your steps to indicate what you are doing. (8 pts.)

$$f(x) = \frac{4x}{3+2x}$$

$$f(x) = 4x \cdot \frac{1}{3+2x} = \frac{4x}{3} \cdot \frac{1}{1+\frac{2x}{3}} = \frac{4x}{3} \cdot \sum_{n=0}^{\infty} \left(\frac{2x}{3}\right)^n$$

$$= \frac{4x}{3} \cdot \sum_{n=0}^{\infty} \frac{2^n \cdot x^n}{3^n} = \boxed{\sum_{n=0}^{\infty} \frac{2^{n+2} \cdot x^{n+1}}{3^{n+1}}}$$

2. Find a degree $N = 3$ Taylor polynomial approximation for the function $f(x) = \ln(x+1)$ written in powers of x. (4 pts.)

$f(x) = \ln(x+1)$ @ $x = 0$ $\quad f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{x+1}$ " $\quad f'(0) = \frac{1}{0+1} = 1$

$f''(x) = \frac{-1}{(x+1)^2}$ " $\quad f''(0) = \frac{-1}{(0+1)^2} = -1$

$f'''(x) = \frac{2}{(x+1)^3}$ $\quad f'''(0) = \frac{2}{(0+1)^3} = 2$

So $\boxed{f(x) \approx x - \frac{x^2}{2!} + \frac{2x^3}{3!}}$

3. **(a)** Find the Taylor series of $\cos(t^3)$ centered at $x = 0$. You may use the fact, without justification, that (2 pts.)

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}.$$

$$\cos(t^3) = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} t^{6n}}$$

(b) Find a series which represents the value of the definite integral $\int_0^1 \cos(t^3)\, dt$ using your answer from part *(a)* above. (6 pts.)

$$\int_0^1 \cos(t^3)\, dt = \int_0^1 \left(\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} t^{6n} \right) dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{t^{6n+1}}{6n+1} \Bigg|_0^1$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{1}{6n+1}}$$