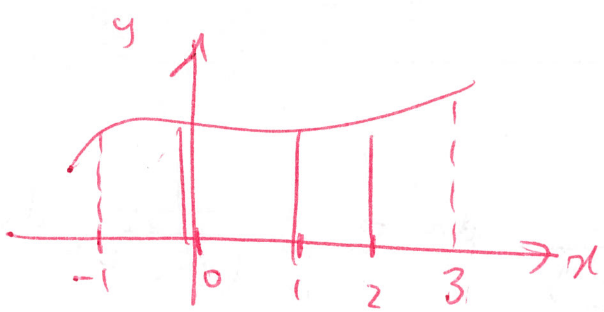
1. (a) Using the FTC: (8 pts.)

$$\frac{d}{dx}\left[\int_{a(x)}^{b} f(t)\,dt\right] = \boxed{-f(a(x))\cdot a'(x)}$$

(b) If F is an antiderivative of f, then: (7 pts.)

$$\int f(g(x))g'(x)\,dx = \boxed{F(g(x)) + C}$$

2. Consider the function $f(x) = -2x^2 + 4x + 6$. Using a left-endpoint estimate with $n = 4$ subintervals, estimate the *average value* of f over the interval $[-1, 3]$. (15 pts.)



$n = 4$

$L_4 = 1 * (0 + 6 + 8 + 6)$

$= \boxed{20}$ ← approx area

$f(-1) = -2(-1)^2 + 4(-1) + 6$

$= -2 - 4 + 6 = \boxed{0}$

$f(0) = \boxed{6}$

$f(1) = -2(1)^2 + 4(1) + 6$

$= \boxed{8}$

$f(2) = -2(2)^2 + 4(2) + 6$

$= \boxed{16}$

$\frac{1}{4} * 20 = \boxed{5}$

approx avg. value

3. Evaluate the integrals. (10 pts. each)

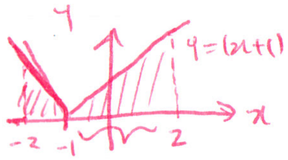
(a) $\int_{-2}^{2} |x+1|\,dx = \int_{-2}^{-1} |x+1|\,dx + \int_{-1}^{2} |x+1|\,dx$

$= \int_{-2}^{-1} -(x+1)\,dx + \int_{-1}^{2} (x+1)\,dx = -\frac{1}{2}x^2 - x\Big|_{-2}^{-1} + \frac{1}{2}x^2 + x\Big|_{-1}^{2}$

$= [-\frac{1}{2}(-1)^2 - (-1)] - [-\frac{1}{2}(-2)^2 - (-2)] + (\frac{1}{2}(2)^2 + (2)] - [\frac{1}{2}(-1)^2 + (-1)]$

$= \cancel{-\frac{1}{2}} \cancel{+1} + \cancel{2} - \cancel{2} + 2 + 2 - \cancel{\frac{1}{2}} + 1 = \boxed{5}$

OR



$\frac{1}{2} + \frac{9}{2} = \boxed{5}$

(b) $\int \frac{\cos(-3x) + \sin(-3x)}{\cos(-3x) - \sin(-3x)}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \boxed{\frac{1}{3}\ln|\cos(-3x) - \sin(-3x)| + C}$

u-sub Box

$u = \cos(-3x) - \sin(-3x)$

$du = -3(-\sin(-3x)) - (-3)\cos(-3x)\,dx$

$du = 3\sin(-3x) + 3\cos(-3x)\,dx$

(c) $\int \frac{1}{\sqrt{9 - (x+2)^2}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1 - (\frac{x+2}{3})^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,du$

u-sub Box

$u = \left(\frac{x+2}{3}\right)$

$du = \frac{1}{3}\,dx$

$= \sin^{-1}(u) + C$

$= \boxed{\sin^{-1}\left(\frac{x+2}{3}\right) + C}$

(d) $\int (1 + \ln x) \sec^2(x \ln x)\, dx$ $= \int \sec^2(u)\, du$

$u = x \ln x$

$du = 1 \cdot \ln x + x \cdot \frac{1}{x}\, dx$

$du = \ln x + 1\, dx$

$= \tan(u) + C$

$= \boxed{\tan(\ln x + 1) + C}$

4. Express the following limit as a definite integral. (8 pts.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n} \ln\left(2 + \frac{5i}{n}\right)$$

$\Delta x = \frac{5}{n}$

$x_i = 2 + \frac{5i}{n}$

$a = x_0 = 2$

$b = x_n = 2 + 5 = 7$

$f(x) = \ln(x)$

$$\boxed{\int_2^7 \ln(x)\, dx}$$

5. Find a closed formula for the right-endpoint Riemann sum of the approximate area under $y = x^2$ over the interval $[0, 1]$ using n rectangles. You do not need to take a limit, but simplify your answer which should be a function of n. (8 pts.)

$\Delta x = \frac{1}{n}$

$x_i = 0 + \frac{i}{n}$

$f(x) = x^2$

$R_n = \sum_{i=1}^{n} \Delta x \cdot f(x_i)$

$= \sum_{i=1}^{n} \frac{1}{n} \left(\frac{i}{n}\right)^2$

$= \frac{1}{n^3} \sum_{i=1}^{n} i^2$

$= \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \boxed{\frac{(n+1)(2n+1)}{6n^2}}$

6. Find the area bounded by the curves $y = -x(x+3)(x-2)$ and $y = -3(x+3)$. (14 pts.)

$y = -3(x+3)$

$y = -x(x+3)(x-2)$

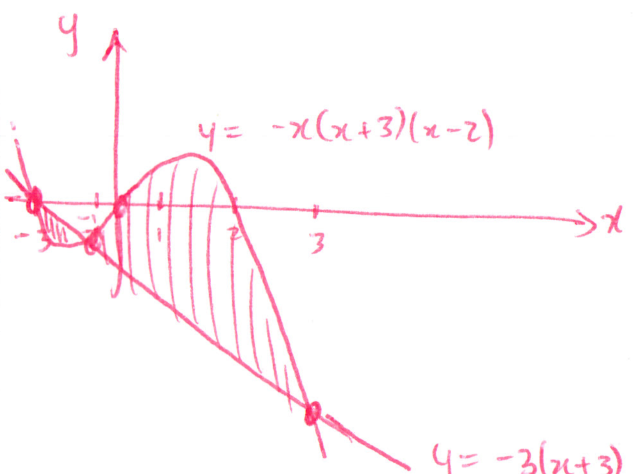
Set $y = y$ $\quad -3\cancel{(x+3)} = -x\cancel{(x+3)}(x-2)$

$\Rightarrow -3 = -x(x-2)$ OR $x = -3$

$\Rightarrow x^2 - 2x - 3 = 0$ OR $x = -3$

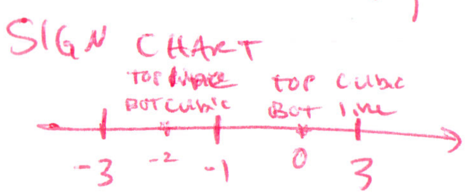
$\Rightarrow (x-3)(x+1) = 0$ OR $x = -3$

$\Rightarrow \boxed{x = 3, -1 \quad \text{OR } x = -3}$



$$\int_{-3}^{-1} \left(-3(x+3) - (-x)(x+3)(x-2)\right) dx$$

$$+ \int_{-1}^{3} \left((-x)(x+3)(x-2) - (-3)(x+3)\right) dx = \ldots$$



$$= \frac{20}{3} + \frac{128}{3} = \boxed{\frac{148}{3}}$$

Bonus: (5 pts.) Find the third derivative evaluated at $x = 0$, $F'''(0)$ for the function $F(x) = \int_0^x e^{t^2}\,dx$.

$F(x) = \int_0^x e^{t^2}\,dt$

$F'(x) = e^{x^2}$

$F''(x) = 2xe^{x^2}$

$F'''(x) = 2e^{x^2} + 4x^2e^{x^2}$

$F'''(0) = 2e^0 + 4 \cdot 0 e^0 = 2 \cdot 1 + 0$

$= \boxed{2}$