Math 1552 1:55 Version B **Name (Print):** ______________________
Spring 2019
Midterm 2 **Canvas email:** ______________________
March 8, 2019
Time Limit: 50 Minutes **TA:** ______________________

GT ID: | | | | | | | | | |

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: ______________________

This exam contains 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.
You are required to show your work on each problem on this exam. The following rules apply:

- **This exam is double-sided.**
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Please circle or box in your final answer.
- If you need extra space, you may use the back side of this cover page.

Problem	Points	Score
1	27	
2	10	
3	15	
4	30	
5	18	
Total:	100	

SCRATCH WORK

If you want work on this page to be graded, please make a note on the corresponding problem.

VERSION B 1:55 Name: ______________________________

1. Let
$$a_n = \left(\frac{n}{n+8}\right)^n.$$

(a) (15 points) Evaluate $\lim_{n\to\infty} a_n$.

Grading Rubric:

- (5 points) Rewrite the equation, or take the natural log of the original function.
$$\lim_{n\to\infty}\left(\frac{n}{n+8}\right)^n = \lim_{n\to\infty}\left(\frac{1}{\frac{n+8}{n}}\right)^n$$
- (5 points) Rewrite again in terms of the formula we know, or use L'Hopital's rule if you started by taking natural logs.
$$= \lim_{n\to\infty}\left(\frac{1}{1+\frac{8}{n}}\right)^n = \lim_{n\to\infty}\left(\frac{1}{\left(1+\frac{8}{n}\right)^n}\right)$$
- (3 points) Evaluate this limit, and (2 points) simplify, or if you used L'Hopital's rule, take e to the limit.
$$= \frac{1}{e^8}.$$

(b) (6 points) Does the sequence $\{a_n\}$ converge or diverge? Explain your answer.

(3 points) Yes, the sequence **CONVERGES** because (3 points) there is finite limit of e^{-8} as $n \to \infty$.

(c) (6 points) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain your answer.

(3 points) No, the series **DIVERGES** because (3 points) the limit of the terms is e^{-8}, which is not equal to 0.

2. **True or False?** Determine if each statement below is always true or sometimes false. Circle your answer to each statement.

   (a) (2 points) FALSE The integral $\int \frac{dx}{(x^2+1)(x^2+4)}$ cannot be evaluated using the method of partial fractions.

   (b) (2 points) TRUE The integral $\int x^3\sqrt{1+x^2}\ dx$ can be evaluated by trigonometric substitution by setting $x = \tan\theta$.

   (c) (2 points) FALSE The integral $\int_{-1}^{1} \frac{1}{x^4}\ dx$ can be evaluated using the Fundamental Theorem of Calculus.

   (d) (2 points) FALSE If $\{a_n\}$ diverges, then $\lim_{n\to\infty} a_n = \infty$.

   (e) (2 points) TRUE If a sequence $\{a_n\}$ does not have a lower bound, then $\{a_n\}$ diverges.

3. (15 points) Evaluate the improper integral

$$\int_5^\infty \frac{20x}{(x^2-9)^{3/2}}\, dx.$$

Grading Rubric:

- (2 points) Rewrite the integral as a limit.

$$\int_5^\infty \frac{20x}{(x^2-9)^{3/2}}\, dx = \lim_{b\to\infty} \int_5^b \frac{10x}{(x^2-9)^{3/2}}\, dx$$

- (3 points) Find u and du, sub into equation. Either change the limits of integration or leave them off.

$$= 10 \lim_{b\to\infty} \int_{16}^{b^2-9} u^{-3/2} du \quad [u = x^2 - 9, du = 2xdx]$$

- (4 points) Find the anti-derivative.

$$= 10 \lim_{b\to\infty} \left[-2u^{-1/2}\right]_{16}^{b^2-9}$$

- (2 points) Plug in values.

$$= -20 \lim_{b\to\infty} \left[(b^2-9)^{-1/2} - 16^{-1/2}\right]$$

- (2 points) Evaluate the limits.

$$= -20 \left[0 - \frac{1}{4}\right]$$

- (2 points) Final answer, ok if not fully simplified: $= 5$.

4. Evaluate each integral using any technique we have learned.

   (a) (15 points)

$$\int \frac{5dx}{\sqrt{25x^2 - 16}}$$

Grading Rubric:

- (2 points) Trig sub: let $5x = 4\sec\theta$, so $x = \frac{4}{5}\sec\theta$.
- (1 point) Find $dx = \frac{4}{5}\sec\theta\tan\theta d\theta$.
- (2 points) Note that $\sqrt{25x^2 - 16}$ reduces to $\sqrt{16\sec^2\theta - 16} = 4\tan\theta$.
- (1 point) Rewrite the integral:

$$\int \frac{5dx}{\sqrt{25x^2 - 16}} = \int \frac{5 \cdot \frac{4}{5}\sec\theta\tan\theta d\theta}{4\tan\theta}$$

- (2 points) Reduce the integral:

$$= \int \sec\theta d\theta$$

- (2 points) Integrate and (1 point) for "+C":

$$= \ln|\sec\theta + \tan\theta| + C$$

- (4 points) Rewrite as a function of x:

$$= \ln\left|\frac{5x}{4} + \frac{\sqrt{25x^2 - 16}}{4}\right| + C.$$

(b) (15 points)

$$\int \frac{x^2+2x+6}{(x^2+4)(x+1)}\,dx$$

- (2 points) Set up equation

$$\frac{x^2+2x+6}{(x^2+4)(x+1)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+1}$$

- (1 points) Combining the right hand side:

$$\frac{x^2+2x+6}{(x^2+4)(x+1)} = \frac{(Ax+B)(x+1)+C(x^2+4)}{(x^2+4)(x+1)}$$

- (2 points) Collect common terms:

$$\frac{x^2+2x+6}{(x^2+4)(x+1)} = \frac{(A+C)x^2+(A+B)x+B+4C}{(x^2+1)(x-2)}$$

- (2 points) Set up the linear system:

$$\begin{cases} A+C=1 \\ A+B=2 \\ B+4C=6 \end{cases}$$

- (3 points) Solve the linear system: $A=0$, $B=2$, $C=1$.
- (4 points) Integrate and (1 point) "+C":

$$\int \left[\frac{1}{x+1} - \frac{2}{x^2+4}\right] dx = \ln|x+1| + \arctan\left(\frac{x}{2}\right) + C$$

5. Answer the following questions given that

$$a_n = \frac{5^n}{6^{n-1}} \quad \text{and} \quad b_n = \frac{3^{2n}}{6^{n-1}}.$$

(a) (6 points) Evaluate $\sum_{n=1}^{\infty} a_n$, or explain why the series diverges.

- (2 points) Rewrite in standard form.

$$\sum_{n=1}^{\infty} \frac{5^n}{6^{n-1}} = 6 \sum_{n=1}^{\infty} \left(\frac{5}{6}\right)^n$$

- (2 point) Use the formula for a geometric series and (1 point) multiply by the first term since n starts at 1. (1 point) for the final answer.

$$= 6 \cdot \frac{5}{6} \cdot \frac{1}{1 - 5/6} = 6 \cdot \frac{5}{6} \cdot 6 = 30.$$

(b) (6 points) Evaluate $\sum_{n=1}^{\infty} b_n$, or explain why the series diverges.
Grading Rubric:

- (1 point) Simplify the expression for a_n:

$$\sum_{n=1}^{\infty} \frac{3^{2n}}{6^{n-1}} = \sum_{n=1}^{\infty} \frac{9^n}{6^{n-1}}$$

- (2 points) Rewrite as a series in standard geometric form:

$$= 6 \sum_{n=1}^{\infty} \left(\frac{9}{6}\right)^n$$

- (2 point) Note that this series **DIVERGES**.
- (1 point) Reason for divergence is because the series is geometric with $r = \frac{9}{6} = \frac{3}{2} > 1$.

(c) (6 points) Does $\sum_{n=1}^{\infty} \frac{a_n}{b_n}$ converge? If so, find the sum. If not, explain why the series diverges.
Grading Rubric:

- (3 points) Divide the terms and simplify to standard form.

$$\sum_{n=1}^{\infty} \frac{5^n}{6^{n-1}} \cdot \frac{6^{n-1}}{3^{2n}} = \sum_{n=1}^{\infty} \left(\frac{5}{9}\right)^n$$

- (1 point) Use the standard formula for geometric series and (1 point) multiply by the first term. (1 point) for final answer.

$$= \frac{5}{9} \cdot \frac{1}{1 - 5/9} = \frac{5}{9} \cdot \frac{9}{4} = \frac{5}{4}.$$