

Math 1552  
Summer 2025  
Exam 1 \*QUP only\*  
June 8 12:00pm – June 9 12:29pm  
Time limit: 75 Minutes

Name (Print): \_\_\_\_\_

Canvas email: \_\_\_\_\_

Teaching Assistant/Section: \_\_\_\_\_

Key

GT ID: 

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Sign Your Name: \_\_\_\_\_

### Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- **Place a box** around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

QUP

1. (2 points) Suppose that  $f(x)$  is a **decreasing** function which is non-negative and not constant on the interval  $[3, 5]$  and  $\int_3^5 f(x) dx = A$ . Which of the following statements are true? *You do not have to show work on this problem.*

true      false

- If we estimate  $A$  using the left-endpoint method with  $n = 4$  rectangles, then we will get a value smaller than  $A$ .
- If we estimate  $A$  using the lower-sum method with  $n = 4$  rectangles, then we will get a value smaller than  $A$ .

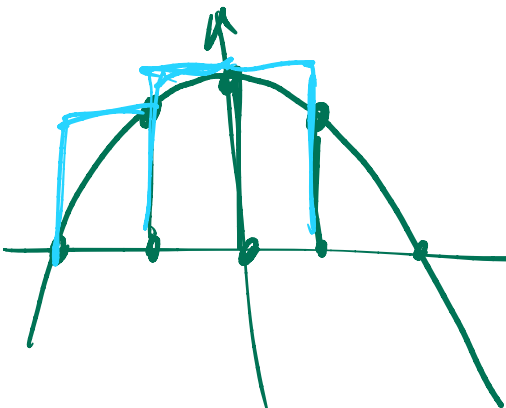
2. (3 points) Suppose  $f(x)$  is an even function and  $g(x)$  is an odd function. If  $\int_{-2}^0 f(x) dx = 6$ ,  $\int_0^3 g(x) dx = 3$ , and  $\int_2^3 g(x) dx = 1$ , find  $A = \int_0^2 f(x) + 2g(x) dx$ .

$$\begin{aligned} \int_0^2 f(x) + 2g(x) dx &= \int_0^2 f(x) dx + 2 \int_0^2 g(x) dx \\ &= 6 + 2 \left( \int_0^3 g(x) dx - \int_2^3 g(x) dx \right) \\ &= 6 + 2(3 - 1) = 6 + 4 = 10 \end{aligned}$$

$A = 10$

3. (3 points) Find the **upper-sum** Riemann sum estimate for the area between the  $x$ -axis and the function  $f(x) = 4 - x^2$  over the interval  $[-2, 1]$  using  $n = 3$  rectangles. area  $\approx$

$11$



$$\begin{aligned} f(-2) &= 4 - (-2)^2 = 0 \\ f(-1) &= 4 - (-1)^2 = 3 \\ f(0) &= 4 \\ f(1) &= 3 \end{aligned}$$

$3 + 4 + 4$

4. (4 points) Compute  $F'(x)$  using the fundamental theorem of calculus.

$$F(x) = \int_0^{x^3} \frac{1-t}{1+t} dt$$

$$G(x) = \int_0^x \frac{1-t}{1+t} dt$$

$$G'(x) = \frac{1-x}{1+x} \text{ by FTC}$$

$$F(x) = G(x^3)$$

$$\Rightarrow F'(x) = G'(x^3) \cdot (x^3)'$$

$$= \frac{1-x^3}{1+x^3} \cdot 3x^2$$

5. (4 points) Find the general antiderivative.

$$\int \frac{2}{4+x^2} dx = \int \frac{1}{2} \cdot \frac{1}{1+(\frac{x}{2})^2} dx$$

$$= \int \frac{1}{1+u^2} du$$

$$= \tan^{-1}(u) + C$$

$$= \tan^{-1}\left(\frac{x}{2}\right) + C$$

u-sub Box

$$u = \frac{x}{2}$$
$$du = \frac{1}{2} dx$$

6. (24 points) Integrate.

$$(a) \int_0^{\sqrt{5}} 2x\sqrt{x^2+4} dx = \int_{*}^{*} \sqrt{u} du$$

u-sub box

$$\begin{aligned} u &= x^2 + 4 \\ du &= 2x dx \end{aligned}$$

$$= \frac{2}{3} u^{3/2} \Big|_{*}^{*}$$

$$= \frac{2}{3} (x^2+4)^{3/2} \Big|_0^{\sqrt{5}}$$

$$= \frac{2}{3} (9^{3/2}) - \frac{2}{3} 4^{3/2} = 18 - \frac{16}{3}$$

$$= \frac{54-16}{3} = \boxed{\frac{38}{3}}$$

$$(b) \int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

Hint: u-sub with  $u = e^x$ .

u-sub box

$$\begin{aligned} u &= e^x \\ du &= e^x dx \end{aligned}$$

$$\int \frac{e^x}{\sqrt{1-(e^x)^2}} dx = \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \sin^{-1}(u) + C$$

$$= \boxed{\sin^{-1}(e^x) + C}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$(c) \int \tan^3(x) \sec^4(x) dx = \int \tan^3 x * \underbrace{\sec^2 x}_{(1+\tan^2 x)} * \underbrace{\sec^2 x dx}_{du}$$

u-sub Box

$$u = \tan x \\ du = \sec^2 x dx$$

$$= \int u^3 (1+u^2) \cdot du = \int u^3 + u^5 du$$

$$= \frac{1}{4} u^4 + \frac{1}{6} u^6 + C$$

$$= \frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$$

$$(d) \int x \ln x dx$$

IBP Box

$$u = \ln x \quad dv = x dx \\ du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$$

$$= \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

7. (10 points) Suppose  $f(x) = 5x + 1$ . Use a general Riemann Sum

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*) \Delta x$$

to evaluate the definite integral of  $f(x)$  on the interval  $[-2, 1]$

(a) Find an expression for  $R_n = \sum_{k=1}^n f(x_k^*) \Delta x$  that does not involve sigma's.

(b) Find the exact value by taking the limit of the expression you found in part (a).

Please box your answer for each part and clearly organize your work.

(a)

$$\begin{aligned} R_n &= \sum_{k=1}^n f(x_k) \Delta x = \sum_{k=1}^n \left( 5 \left( -2 + \frac{3k}{n} \right) + 1 \right) \frac{3}{n} \\ &= \sum_{k=1}^n -9 \cdot \frac{3}{n} + \frac{15k}{n^2} \cdot \frac{3}{n} = -27 \sum_{k=1}^n \frac{1}{n} + \frac{45}{n^2} \sum_{k=1}^n k \\ &= -27 + \frac{45}{n^2} \frac{n(n+1)}{2} = \boxed{-27 + 22.5 \frac{n(n+1)}{n^2}} \end{aligned}$$

(b)

$$\begin{aligned} \lim_{n \rightarrow \infty} R_n &= \lim_{n \rightarrow \infty} -27 + 22.5 \frac{n(n+1)}{n^2} = -27 + 22.5 \\ &= \boxed{-4.5} \end{aligned}$$