Math 1552
Summer 2023
Exam 1
June 8 @ 12:30pm
Time limit: 75 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Jal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- **Place a box** around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

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1. (2 points) Suppose that $f(x)$ is an **increasing** function which is non-negative and not constant on the interval $[3,5]$ and $\int_3^5 f(x)\,dx = A$. Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
●	○	If we estimate A using the upper-sum method with $n = 4$ rectangles, then we will get a value larger than A.
○	●	If we estimate A using the right-endpoint method with $n = 4$ rectangles, then we will get a value smaller than A.

2. (3 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-3}^0 f(x)\,dx = 6$, $\int_2^3 f(x)\,dx = 2$, and $\int_{-2}^0 g(x)\,dx = 3$, find $A = \int_0^2 f(x) - g(x)\,dx$.

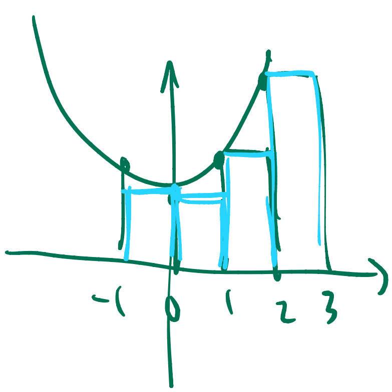
$A =$ $\boxed{7}$

$\int_0^2 f(x) - g(x)\,dx$

$= \int_0^2 f(x)\,dx - \int_0^2 g(x)\,dx$ g odd f even

$= \left[\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx\right] + \int_{-2}^0 g(x)\,dx = [6-2] + 3 = 7$

3. (3 points) Find the **lower-sum** Riemann sum estimate for the area between the x-axis and the function $f(x) = x^2 + 1$ over the interval $[-1,3]$ using $n = 4$ rectangles. area $\approx$ $\boxed{9}$



$f(-1) = 2$] 1
$f(0) = 1$] 1
$f(1) = 2$) 2
$f(2) = 5$) 5
$f(3) = 10$

$1+1+2+5 = 9$

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4. (4 points) Compute $F'(x)$ using the fundamental theorem of calculus.

$$F(x) = \int_1^{x^2} \frac{t^3}{1+t^2}\,dt$$

$G(x) = \int_1^x \frac{t^3}{1+t^2}\,dt$

$G'(x) = \frac{x^3}{1+x^2}$ by FTC

$F(x) = G(x^2)$

$\Rightarrow F'(x) = G'(x^2) * (x^2)'$

$= \frac{x^6}{1+x^2} \cdot 2x = \boxed{\frac{2x^7}{1+x^2}}$

5. (4 points) Find the general antiderivative.

$$\int \frac{3}{9+x^2}\,dx = \int \frac{\cancel{3}}{\cancel{9}_{3}\left(1+\left(\frac{x}{3}\right)^2\right)}\,dx$$

U-Sub Box

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$= \int \frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \frac{1}{3}\,dx$

$= \tan^{-1}(u) + C$

$= \boxed{\tan^{-1}\left(\frac{x}{3}\right) + C}$

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6. (24 points) Integrate.

(a) $\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx = \int_*^* \frac{1}{u^2}\,du = \frac{-1}{u}\Big|_*^*$

u-sub Box

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \frac{-1}{\ln x}\Big|_e^{2e} = \frac{-1}{\ln(2e)} - \frac{-1}{\ln e}$

$= \frac{-1}{2} + \frac{1}{1} = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

(b) $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = \int \frac{1}{\sqrt{u}}\,du$

u-sub Box

$u = x^3 + 1$

$du = 3x^2\,dx$

$= 2\sqrt{u} + C$

$= \boxed{2\sqrt{x^3+1} + C}$

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(c) $\int \sin^3(2x)\cos^2(2x)\,dx$ $= \int \sin(2x)\cdot \underbrace{\sin^2(2x)}_{(1-\cos^2(2x))} \cos^2(2x)\,dx$

$$= \int (1-\cos^2(2x))\cos^2(2x)\sin(2x)\,dx$$

u-sub Box

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$$= \int (1-u^2)u^2 \tfrac{-1}{2}du$$

$$= -\frac{1}{2}\int u^2 - u^4\,du = -\frac{1}{2}\cdot\frac{1}{3}u^3 - \frac{-1}{2}\cdot\frac{1}{5}u^5 + C$$

$$= \boxed{-\frac{1}{6}\cos^3 2x + \frac{1}{10}\cos^5 2x + C}$$

(d) $\int 2x\sec^2(2x)\,dx$

Hint: integration by parts.

IBP Box

$u = 2x$ $\quad dv = \sec^2(2x)\,dx$

$du = 2\,dx$ $\quad v = \frac{1}{2}\tan(2x)$

$$= x\tan(2x) - \int \tan(2x)\,dx$$

$$= x\tan(2x) - \int \frac{\sin(2x)}{\cos(2x)}\,dx$$

u-sub Box

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

$$= x\tan(2x) - \int \frac{1}{u}\cdot\frac{-1}{2}du$$

$$= x\tan(2x) + \frac{1}{2}\ln|u| + C$$

$$= \boxed{x\tan(2x) + \frac{1}{2}\ln|\cos 2x| + C}$$

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7. (10 points) Suppose $f(x) = 3x + 1$. Use a general Riemann Sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_k = -1 + k\Delta x = -1 + \frac{3k}{n}$

to evaluate the definite integral of $f(x)$ on the interval $[-1, 2]$

(a) Find an expression for $R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$ that does not involve sigma's.

(b) Find the exact value by taking the limit of the expression you found in part (a).

Please box your answer for each part and clearly organize your work.

(a)

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left(3\cdot\left(-1+\frac{k\cdot 3}{n}\right)+1\right)\cdot\frac{3}{n}$$

$$= \sum_{k=1}^{n} \left(\frac{3k}{n}-2\right)\frac{3}{n} = \sum_{k=1}^{n} \frac{27k}{n^2} - \frac{6}{n} = \frac{27}{n^2}\sum_{k=1}^{n} k - \frac{6}{n}\sum_{k=1}^{n} 1$$

$$= \frac{27}{n^2}\,\frac{n(n+1)}{2} - \frac{6}{\cancel{n}}\cdot\cancel{n} \qquad \boxed{R_n = 13.5\,\frac{n(n+1)}{n^2} - 6} \text{ (a)}$$

(b) $$\lim_{n\to\infty} R_n = \lim_{n\to\infty} 13.5\left(\frac{n(n+1)}{n^2}\right) - 6 = 13.5 - 6 = \boxed{7.5} \text{ (b)}$$

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