Math 1552
Summer 2023
Exam 1 Practice
June 8
Time limit: 75 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID:

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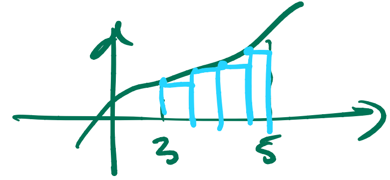
Sign Your Name: ______________________

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- **Place a box** around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.



1. (2 points) Suppose that $f(x)$ is a function which is non-negative on the interval $[3, 5]$ and $\int_3^5 f(x)\,dx = A$. Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
●	○	If we estimate A using the left-endpoint method with $n = 4$ rectangles, then we will always get a value smaller than A.
●	○	If we estimate A using the upper-sum method with $n = 4$ rectangles, then we will get a value at least as large as A.

2. (3 points) Suppose $f(x)$ is an even function and $g(x)$ is an odd function. If $\int_{-2}^{2} f(x)\,dx = 10$, $\int_0^3 g(x)\,dx = 4$, and $\int_2^3 g(x)\,dx = 1$, find $A = \int_0^2 2f(x) - 3g(x)\,dx$.

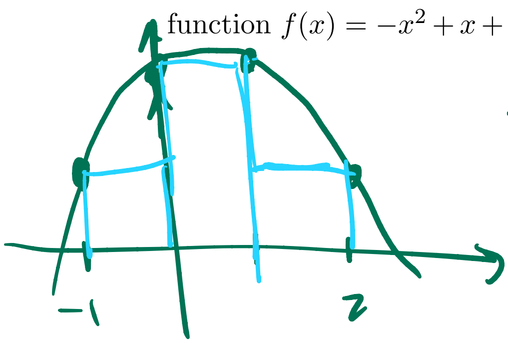
$A =$ 1

$$\int_0^2 2f(x) - 3g(x)\,dx = 2\int_0^2 f(x)\,dx - 3\int_0^2 g(x)\,dx$$

$$= 2(5) - 3\left(\int_0^3 g(x)\,dx - \int_2^3 g(x)\,dx\right)$$

$$= 10 - 3(4-1) = 10 - 3\cdot 3 = 10 - 9 = 1$$

3. (3 points) Find the **lower-sum** Riemann sum estimate for the area between the x-axis and the function $f(x) = -x^2 + x + 3$ over the interval $[-1, 2]$ using $n = 3$ rectangles. area $\approx$ 5



$f(-1) = -(-1)^2 - 1 + 3 = 3 - 2 = 1$) 1

$f(0) = 3$) 3

$f(1) = -1 + 1 + 3 = 3$) 1

$f(2) = -4 + 2 + 3 = 1$

$1 + 3 + 1 = 5$

4. (4 points) Compute $F'(x)$ using the fundamental theorem of calculus.

$$F(x) = \int_0^{\ln x} \sqrt{1-t^2}\, dt$$

$G(x) = \int_0^x \sqrt{1-t^2}\, dt$

$\Rightarrow G'(x) = \sqrt{1-x^2}$ by FTC

$F(x) = G(\ln x)$

$\Rightarrow F'(x) = G'(\ln x) \cdot (\ln x)'$

$= \sqrt{1-(\ln x)^2} \cdot \frac{1}{x}$

$= \boxed{\frac{\sqrt{1-(\ln x)^2}}{x}}$

5. (4 points) Find the general antiderivative.

$$\int \sqrt{x} + \frac{1}{\sqrt{x}} + \frac{1}{2(1+x^2)}\, dx$$

$= \int x^{1/2} + x^{-1/2} + \frac{1}{2} \cdot \frac{1}{1+x^2}\, dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + \frac{1}{2}\tan^{-1}(x) + C$

$= \boxed{\frac{2}{3}x^{3/2} + 2\sqrt{x} + \frac{1}{2}\tan^{-1}(x) + C}$

6. (24 points) Integrate.

(a) $\displaystyle\int_1^2 \frac{2\ln x}{x}\,dx = \int_*^* 2\cdot u\,du = u^2\Big|_*^*$

u-sub Box
$u = \ln x$
$du = \frac{1}{x}\,dx$

$= (\ln x)^2\Big|_1^2$

$= (\ln 2)^2 - (\ln 1)^2$ ($\ln 1 \to 0$)

$= \boxed{(\ln 2)^2}$

(b) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{1}{1+(e^x)^2}\cdot e^x\,dx$

Hint: u-sub with $u = e^x$.

u-sub Box
$u = e^x$
$du = e^x\,dx$

$= \displaystyle\int \frac{1}{1+u^2}\,du$

$= \tan^{-1}(u) + C$

$= \boxed{\tan^{-1}(e^x) + C}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \tan^2 x + 1 = \sec^2 x$

$\Rightarrow \sec^2 x - 1 = \tan^2 x$

$u = \sec x$

(c) $\int \tan^3(x)\sec^3(x)\,dx$ $= \int \tan^2(x)\cdot\sec^2(x)\cdot \sec(x)\tan(x)\,dx$

(replace $\tan^2(x)$ w/ $\sec^2 x - 1$; $\sec(x)\tan(x)\,dx \to du$)

U-sub Box

$u = \sec x$

$du = \sec x \tan x\,dx$

$$= \int (u^2-1)\,u^2\cdot du = \int u^4 - u^2\,du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

(d) $\int x^3\sqrt{x^2+1}\,dx$

Hint: integration by parts.

IBP box

$u = x^2 \quad dv = x\sqrt{x^2+1}\,dx$

$du = 2x\,dx \quad v = \frac{1}{3}(x^2+1)^{3/2}$

u-sub:

$$\int x\sqrt{x^2+1}\,dx$$

U-sub Box: $u = x^2+1$, $du = 2x\,dx$

$$= \frac{1}{2}\cdot\frac{2}{3}(x^2+1)^{3/2} + C$$

$$= \frac{1}{3}(x^2+1)^{3/2} + C$$

$$\int x^3\sqrt{x^2+1}\,dx = \frac{x^2}{3}(x^2+1)^{3/2} - \int 2x\cdot\frac{1}{3}(x^2+1)^{3/2}\,dx$$

U-sub Box: $u = x^2+1$, $du = 2x\,dx$

$$= \frac{x^2}{3}(x^2+1)^{3/2} - \frac{1}{3}\cdot\frac{2}{5}(x^2+1)^{5/2} + C$$

$$= \boxed{\frac{x^2}{3}(x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + C}$$

7. (10 points) Suppose $f(x) = 4x - 1$. Use a general Riemann Sum

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

to evaluate the definite integral of $f(x)$ on the interval $[-1, 2]$.
(a) Find an expression for $R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$ that does not involve sigma's.
(b) Find the exact value by taking the limit of the expression you found in part (a).

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$x_k = a + k\cdot\Delta x = -1 + \frac{3k}{n}$

(a)

Please box your answer for each part and clearly organize your work.

$R_n = \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} \left(4\left(-1+\frac{3k}{n}\right)-1\right)\cdot\frac{3}{n} = \sum_{k=1}^{n} -5\cdot\frac{3}{n} + \frac{12k}{n}\cdot\frac{3}{n}$

$= -\frac{15}{n}\sum_{k=1}^{n} 1 + \frac{36}{n^2}\sum_{k=1}^{n} k = -\frac{15}{n}\cdot n + \frac{36}{n^2}\cdot\frac{n(n+1)}{2}$

$\boxed{R_n = -15 + 18\frac{n(n+1)}{n^2}}$ (a)

(b) $\lim_{n\to\infty} R_n = -15 + 18\cdot 1 = 18 - 15 = \boxed{3}$

Check ans. $\int_{-1}^{2} 4x - 1\, dx = 2x^2 - x \Big|_{-1}^{2}$

$= (2\cdot 4 - 2) - (2+1)$

$= 6 - 3 = \boxed{3}$ ✓

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