Math 1552
Summer 2023
Exam 2 GLC
June 8 @ 3:30pm
Time limit: 75 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: Sal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- $\boxed{\textbf{Place a box}}$ around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

1. (4 points) Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
○	●	If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ converges.
○	●	The telescoping series $\sum_{n=2}^{\infty} \ln\sqrt{n+1} - \ln\sqrt{n}$ converges.
●	○	The series $\sum_{n=2}^{\infty} \frac{n}{(n^2+1)^{3/2}}$ converges by the integral test.
○	●	The series $\sum_{n=1}^{\infty} n^p$ converges when $p = \frac{1}{2}$.

2. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{\ln\left(\left(1-\frac{2}{n}\right)^n\right)\right\}$

$\boxed{-2}$

(b) $\left\{\frac{(-1)^n n}{n^2+1}\right\}$

$\boxed{0}$

3. (10 points) Find the value of the convergent geometric series below.
Hint: note that the starting value is $n = 1$.

$$\sum_{n=1}^{\infty} \frac{5^n + (-1)^n}{3^{2n}}$$

$(r = \frac{5}{9})$ $(r = \frac{-1}{9})$

$$\sum_{n=1}^{\infty} \frac{5^n+(-1)^n}{3^{2n}} = \sum_{n=1}^{\infty} \left(\frac{5}{9}\right)^n + \left(\frac{-1}{9}\right)^n = \sum_{n=1}^{\infty}\left(\frac{5}{9}\right)^n + \sum_{n=1}^{\infty}\left(\frac{-1}{9}\right)^n$$

$$= \frac{5}{9}\sum_{n=0}^{\infty}\left(\frac{5}{9}\right)^n + \frac{-1}{9}\sum_{n=1}^{\infty}\left(\frac{-1}{9}\right)^n$$

$$\stackrel{\star}{=} \frac{5}{9}\left(\frac{1}{1-5/9}\right) + \frac{-1}{9}\left(\frac{1}{1- -1/9}\right)$$

$$= \frac{5}{9}\,\frac{1}{4/9} - \frac{1}{9}\left(\frac{1}{10/9}\right)$$

$$= \frac{5}{\cancel{9}} \cdot \frac{\cancel{9}}{4} - \frac{1}{\cancel{9}} \cdot \frac{\cancel{9}}{10} = \frac{5}{4} - \frac{1}{10}$$

$$= \frac{25}{20} - \frac{2}{20} = \boxed{23/20}$$

$\star$ Since $|r|<1$ when $r = \frac{5}{9}$ & $r = \frac{-1}{9}$ by the geometric series formula

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

4. (10 points) Evaluate the improper integral. If the improper integral diverges write DNE.

$$\int_4^\infty \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx$$

$$\int_4^N \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx = \int_*^* \frac{1}{e^u}\cdot 2\,du = 2\int_*^* e^{-u}\,du$$

$$= -2e^{-u}\Big|_*^*$$

u-sub

$$\boxed{\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \\ 2\,du &= \frac{1}{\sqrt{x}}\,dx \end{aligned}}$$

$$= -2e^{-\sqrt{x}}\Big|_4^N$$

$$= -2e^{-\sqrt{N}} - (-2e^{-\sqrt{4}})$$

$$= 2e^{-2} - 2e^{-\sqrt{N}}$$

So $\displaystyle\int_4^\infty \frac{1}{\sqrt{x}e^{\sqrt{x}}}\,dx = \lim_{N\to\infty} 2e^{-2} - 2e^{-\sqrt{N}} = \frac{2}{e^2} - 0$

$$= \boxed{\frac{2}{e^2}}$$

5. (10 points) Integrate. *Hint: use partial fractions.*

$$\int_1^{e^2} \frac{1}{x(x+1)}\,dx$$

$$\frac{A}{x} + \frac{B}{x+1} = \frac{A(x+1) + Bx}{x(x+1)} = \frac{1}{x(x+1)}$$

$x(A+B) + A = 1.$

So $A+B=0,\ A=1$

So $A=1,\ B=-1.$

$$\int_1^{e^2} \frac{1}{x} - \frac{1}{x+1}\,dx$$

$$= \ln x - \ln(x+1)\Big|_1^{e^2}$$

$$= \underbrace{\ln(e^2)}_{2} - \ln(e^2+1) - (\underbrace{\ln 1}_{0} - \ln(1+1))$$

$$= \underbrace{-\ln(e^2+1) + \ln(2) + 2}_{\text{also ok}} = \boxed{\ln\left(\frac{2}{e^2+1}\right) + 2}$$

6. (10 points) Integrate.

$$\int \frac{1}{(4+x^2)^{3/2}}\,dx = \int \frac{1}{(4+4\tan\theta)^{3/2}}\, 2\sec^2\theta \, d\theta$$

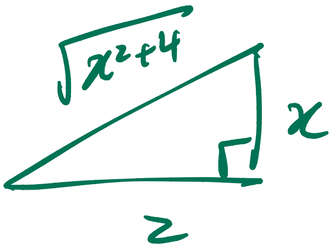
trig sub Box

$$\boxed{\begin{aligned} x &= 2\tan\theta \\ dx &= 2\sec^2\theta\, d\theta \end{aligned}}$$

$$= \int \frac{1}{(4\sec^2\theta)^{3/2}} \cdot 2\sec^2\theta \, d\theta$$

$\tan\theta = x/2 = \frac{\text{opp}}{\text{adj}}$

$$= \int \frac{1}{8\sec^3\theta} \cdot 2\sec^2\theta \, d\theta$$



$$= \frac{1}{4}\int \frac{1}{\sec\theta}\, d\theta = \frac{1}{4}\int \cos\theta \, d\theta$$

$$= \frac{1}{4}\sin\theta + C$$

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$$= \boxed{\frac{1}{4}\frac{x}{\sqrt{x^2+4}} + C}$$

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