Math 1552
Summer 2023
Exam 1 *QUP only*
June 8 12:30pm – June 9 12:29pm
Time limit: 75 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID:

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Sign Your Name: Sal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- **Place a box** around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

1. (4 points) Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
○	●	If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ converges.
●	○	The telescoping series $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$ converges.
○	●	The series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges by the integral test.
●	○	The series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges when $p = 1$.

2. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{ \left(\frac{3+n}{n} \right)^{-n} \right\}$

e^{-3}

(b) $\left\{ \frac{(-1)^n (n^2-1)}{n^2+1} \right\}$

DNE

3. (10 points) Find the value of the convergent geometric series below.
Hint: note that the starting value is $n = 1$.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^{n-1}}{5^{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^{n-1}}{5^{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 4^n}{4 \cdot 5 \cdot 5^n} = \frac{1}{20} \sum_{n=1}^{\infty} \left(\frac{-4}{5}\right)^n = \frac{1}{20} \cdot \frac{-4}{5} \sum_{n=0}^{\infty} \left(\frac{-4}{5}\right)^n$$

$$\star = \frac{-1}{25} \cdot \frac{1}{1-(-4/5)} = \frac{-1}{25} \cdot \frac{1}{1+4/5} = \frac{-1}{25} \cdot \frac{1}{9/5}$$

$$= \frac{-1}{25} \cdot \frac{5}{9} = \boxed{\frac{-1}{45}}$$

Since $|r|<1$ when $r = -4/5$ we have

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}.$$

4. (10 points) Evaluate the improper integral. If the improper integral diverges write DNE.

$$\int_2^\infty \frac{9x^2}{(x^3+1)^{5/2}}$$

$$\int_2^N \frac{9x^2}{(x^3+1)^{5/2}}\,dx = \int_*^{*} \frac{9}{u^{5/2}}\,\frac{1}{3}\,du = 3\cdot\frac{u^{-3/2}}{-3/2}\Bigg|_*^{*}$$

$u = x^3+1$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$$= 3\cdot\frac{-2}{3}\cdot(x^3+1)^{-3/2}\Big|_2^N$$

$$= -2\left(N^3+1\right)^{-3/2} + 2\left(2^3+1\right)^{-3/2}$$

$$= \frac{2}{27} - \frac{2}{(N^3+1)^{3/2}}$$

So $$\int_2^\infty \frac{9}{(x^3+1)^{5/2}}\,dx = \lim_{N\to\infty}\frac{2}{27} - \frac{2}{(N^3+1)^{3/2}} = \frac{2}{27} - 0$$

$$= \boxed{\frac{2}{27}}$$

5. (10 points) Integrate. *Hint: use partial fractions.*

$$\int_{e-2}^{e-1} \frac{3x+1}{(x+1)(x+2)}\,dx = \int_{e-2}^{e-1} \frac{-2}{x+1} + \frac{5}{x+2}\,dx$$

$$\frac{3x+1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$A(x+2) + B(x+1) = 3x+1$$

$$\Rightarrow (A+B)x + (2A+B) = 3x+1$$

$$\Rightarrow \begin{aligned} 2A+B &= 1 \\ A+B &= 3 \end{aligned}$$

$$\Rightarrow A = 3-B$$

$$\Rightarrow 2(3-B)+B = 1$$

$$B = 5$$

$$A = -2$$

$$= -2\ln|x+1| + 5\ln|x+2| \Big|_{e-2}^{e-1}$$

$$= -2\ln(e-1+1) + 5\ln(e-1+2) - \left(-2\ln(e-2+1) + 5\ln(e-2+2)\right)$$

$$= -2\ln(e) + 5\ln(e+1) + 2\ln(e-1) - 5\ln(e)$$

$$= \boxed{5\ln(e+1) + 2\ln(e-1) - 7}$$

6. (10 points) Integrate.

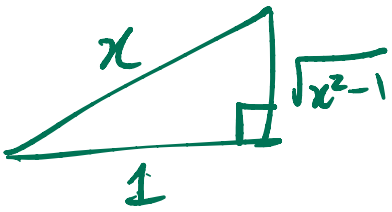
$$\int \frac{1}{x^4\sqrt{x^2-1}}\,dx$$

trig sub Box

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sec\theta = x = \frac{hyp}{adj}$



$\sin\theta = \frac{opp}{hyp}$

u-sub Box

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \int \frac{1}{\sec^4\theta\sqrt{\sec^2\theta-1}} \cdot \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{\sec^3\theta\cdot\tan\theta} \cdot \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{1}{\sec^3\theta}\,d\theta = \int \cos^3\theta\,d\theta$$

$$= \int \cos^2\theta\cdot\cos\theta\,d\theta$$

$$= \int (1-\sin^2\theta)\cos\theta\,d\theta$$

$$= \int 1-u^2\,du$$

$$= u - \frac{1}{3}u^3 + C$$

$$= \sin\theta - \frac{1}{3}\sin^3\theta + C$$

$$= \boxed{\frac{\sqrt{x^2-1}}{x} - \frac{1}{3}\frac{(x^2-1)^{3/2}}{x^3} + C}$$

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