

Math 1552
Summer 2023
Exam 1
June 8 @ 12:30pm
Time limit: 75 Minutes

Name (Print): _____
Canvas email: _____
Teaching Assistant/Section: _____

Key

GT ID:

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By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: _____
Sal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- **Place a box** around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

1. (4 points) Which of the following statements are true? *You do not have to show work on this problem.*

true false

- If the series $\sum_{n=1}^{\infty} a_n$ diverges, then $\lim_{n \rightarrow \infty} a_n \neq 0$.
- The telescoping series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ converges.
- The series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges by the integral test.
- The series $\sum_{n=1}^{\infty} n^p$ converges when $p = -2$.
-

2. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{ \left(1 - \frac{1}{n}\right)^{2n} \right\}$

e^{-2}

(b) $\left\{ \frac{(-1)^n 3}{2n+1} \right\}$

0

3. (10 points) Find the value of the convergent geometric series below.
Hint: note that the starting value is $n = 1$.

$$\sum_{n=1}^{\infty} \frac{3^{n+1}}{2^{2n}} = \sum_{n=1}^{\infty} \frac{3 \cdot 3^n}{(2^2)^n} = 3 \cdot \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n = 3 \cdot \frac{3}{4} \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n$$
$$= 3 \cdot \frac{3}{4} \cdot \left(\frac{1}{1 - 3/4}\right) = 3 \cdot \frac{3}{4} \cdot \frac{1}{1/4} = 3 \cdot \frac{3}{4} \cdot 4 = 9$$

★ Since $|r| < 1$ when $r = 3/4$ by the geometric series formula

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{we have}$$

4. (10 points) Evaluate the improper integral. If the improper integral diverges write DNE.

$$\int_2^{\infty} x e^{-x} dx$$

$$\int_2^N x e^{-x} dx = -x e^{-x} \Big|_2^N + \int_2^N +e^{-x} dx$$

IBP

$$\boxed{\begin{array}{l} u = x \quad dv = e^{-x} dx \\ du = dx \quad v = -e^{-x} \end{array}}$$

$$= -N e^{-N} - (2 e^{-2}) + (-e^{-x} \Big|_2^N)$$

$$= -N e^{-N} + \frac{2}{e^2} + (-e^{-N} - (-e^{-2}))$$

$$= -N e^{-N} + \frac{2}{e^2} + \frac{1}{e^2} - e^{-N}$$

$\underbrace{\qquad\qquad\qquad}_{=3/e^2}$

$$\text{So } \int_2^{\infty} x e^{-x} dx = \lim_{N \rightarrow \infty} \underbrace{-N e^{-N}}_{\star} + \frac{3}{e^2} - e^{-N}$$

$$= 0 + \frac{3}{e^2} - 0 = \boxed{\frac{3}{e^2}}$$

(*) Note $\lim_{N \rightarrow \infty} N e^{-N}$

$$= \lim_{N \rightarrow \infty} \frac{2}{e^2} = \lim_{N \rightarrow \infty} \frac{1}{e^2} = 0$$

\uparrow
L'Hôpital's rule

5. (10 points) Integrate. Hint: use partial fractions.

$$\int_1^5 \frac{x+2}{x^2+4x} dx = \int_1^5 \frac{1/2}{x} + \frac{1/2}{x+4} dx = \left. \frac{1}{2} \ln x + \frac{1}{2} \ln(x+4) \right|_1^5$$

$$\frac{1}{x^2+4x} = \frac{1}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4} = \frac{1}{2} \ln 5 + \frac{1}{2} \ln 9 - \left(\frac{1}{2} \ln(1) + \frac{1}{2} \ln(5) \right)$$

$$A(x+4) + Bx = x+2$$

$$\Rightarrow (A+B)x + 4A = x+2$$

$$\Rightarrow A+B=1$$

$$4A=2$$

$$\Rightarrow A=1/2, B=1/2$$

$$= \frac{1}{2} \ln 9$$

$$= \ln(9^{1/2})$$

$$= \boxed{\ln(3)}$$

6. (10 points) Integrate.

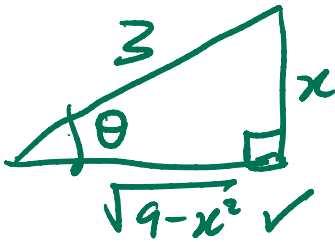
$$\int \frac{1}{x^2 \sqrt{9-x^2}} dx = \int \frac{1}{9 \sin^2 \theta \sqrt{9-9 \sin^2 \theta}} \cdot 3 \cos \theta d\theta = \int \frac{\cancel{3 \cos \theta}}{9 \sin^2 \theta \cdot \cancel{\sqrt{9 \cos^2 \theta}}} d\theta$$

Trig sub

$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$\frac{x}{3} = \sin \theta$$



"basic"
 $\cot \theta = \frac{\text{adj}}{\text{opp}}$

$$= \frac{1}{9} \int \csc^2 \theta d\theta$$



$$= \frac{1}{9} (-\cot \theta) + C$$

$$= -\frac{1}{9} \cot \theta + C$$

$$= \boxed{-\frac{1}{9} \frac{\sqrt{9-x^2}}{x} + C}$$

★

SAVITY CHECK: what's $\int \csc^2 \theta d\theta$?

We know:

$$\int \sec^2 \theta d\theta = \tan \theta + C \checkmark$$

Guess:

$$\int \csc^2 \theta d\theta = -\cot \theta + C?$$

Check

$$(\cot \theta)' = \left(\frac{\cos \theta}{\sin \theta} \right)'$$

$$= \frac{\sin \theta \cdot (\cos \theta)' - \cos \theta (\sin \theta)'}{\sin^2 \theta}$$

$$= \frac{-\sin \theta - \cos^2 \theta}{\sin^2 \theta}$$

$$= -\frac{1}{\sin^2 \theta} = -\csc^2 \theta \checkmark$$

This page may be used for scratch work. Please indicate clearly on the problem if you would like your work on this page to be graded. Loose scrap paper is not permitted.
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