Math 1552
Summer 2023
Exam 1 Practice

Time limit: 75 Minutes

Name (Print): Key

Canvas email:

Teaching Assistant/Section:

GT ID: | | | | | | | | |

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Sign Your Name: Sal

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- $\boxed{\textbf{Place a box}}$ around your final answer for full credit.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 5 pages of questions.

1. (4 points) Which of the following statements are true? *You do not have to show work on this problem.*

true	false	
●	○	If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
○	●	The telescoping series $\sum_{n=1}^{\infty} \sqrt{n+2} - \sqrt{n+1}$ converges.
●	○	The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)^2}$ converges by the integral test.
○	●	The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^p}}$ converges when $p = 2$.

2. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{ \ln\left(\left(1 + \frac{2}{n}\right)^n \right) \right\}$

$\boxed{2}$

(b) $\left\{ \frac{(-1)^n (n+3)!}{(n+1)!} \right\}$

$\boxed{\text{DNE}}$

3. (10 points) Find the value of the convergent geometric series below.
Hint: note that the starting value is $n = 1$.

$$\sum_{n=1}^{\infty} \frac{4^{n-1} - (-1)^n}{3^{2n+1}} \qquad \left(r = \tfrac{4}{9}\right) \qquad \left(r = \tfrac{-1}{9}\right)$$

$$\sum_{n=1}^{\infty} \frac{4^n \cdot 4^{-1}}{3^{2n} \cdot 3} - \sum_{n=1}^{\infty} \frac{(-1)^n}{3^{2n} \cdot 3} = \frac{1}{12} \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n - \frac{1}{3} \sum_{n=1}^{\infty} \left(\frac{-1}{9}\right)^n$$

$$\overset{\star}{=} \frac{1}{\cancel{12}\,3} \cdot \frac{\cancel{4}}{9} \cdot \frac{1}{1-4/9} - \frac{1}{3} \cdot \frac{-1}{9} \cdot \frac{1}{1-\left(-\frac{1}{9}\right)} = \frac{1}{27} \cdot \frac{1}{5/9} + \frac{1}{27} \cdot \frac{1}{10/9}$$

$$= \frac{1}{\cancel{27}\,3} \cdot \frac{\cancel{9}}{5} + \frac{1}{\cancel{27}\,3} \cdot \frac{\cancel{9}}{10} = \frac{1}{15} + \frac{1}{30} = \frac{2+1}{30} = \frac{3}{30}$$

$$= \boxed{\frac{1}{10}}$$

$\star$ Since $|r| < 1$ when $r = \frac{4}{9}$ & $r = \frac{-1}{9}$ by the geometric series formula

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{we have}$$

4. (10 points) Evaluate the improper integral. If the improper integral diverges write DNE.

$$\int_{e^2}^{\infty} \frac{1}{x(\ln x)^2}\,dx$$

$$\int_{e^2}^{N} \frac{1}{x(\ln x)^2}\,dx = \int_{*}^{*} \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_{*}^{*} = \frac{-1}{\ln x}\Big|_{e^2}^{N}$$

U-Sub Box

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}}$$

$$= \frac{-1}{\ln N} - \frac{-1}{\cancel{\ln(e^2)}\;2}$$

$$= \frac{1}{2} - \frac{1}{\ln N}$$

So $$\int_{e^2}^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{N\to\infty} \frac{1}{2} - \frac{1}{\ln N} = \frac{1}{2} - 0$$

$$= \boxed{\frac{1}{2}}$$

5. (10 points) Integrate. *Hint: use partial fractions.*

$$\int_1^4 \frac{3}{x^2+2x}\,dx = \int_1^4 \frac{3/2}{x} - \frac{3/2}{x+2}\,dx$$

$$\frac{3}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$\Rightarrow A(x+2) + Bx = 3$$

$$\Rightarrow x(A+B) + 2A = 3$$

$$\Rightarrow A+B=0$$
$$2A=3$$

$$\Rightarrow A = 3/2,\ B = 3/2$$

$$= \frac{3}{2}\ln x - \frac{3}{2}\ln(x+2)\Big|_1^4$$

$$= \frac{3}{2}\ln 4 - \frac{3}{2}\ln(6) - \left[\cancel{\frac{3}{2}\ln(1)}^{\,0} - \frac{3}{2}\ln(3)\right]$$

$$= \frac{3}{2}\ln 4 + \frac{3}{2}\ln 3 - \frac{3}{2}\ln(6)$$

$$= \frac{3}{2}\left(\ln 4 + \ln 3 - \ln 6\right)$$

$$= \frac{3}{2}\ln\left(\frac{4\cdot 3}{6}\right)$$

$$= \boxed{\frac{3}{2}\ln(2)}$$

6. (10 points) Integrate.

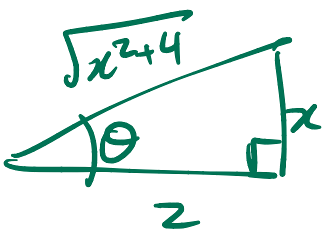
$$\int \frac{x^3}{\sqrt{x^2+4}}\,dx = \int \frac{8\tan^3\theta}{\sqrt{4\tan^2\theta+4}}\cdot 2\sec^2\theta\,d\theta$$

trig sub Box

$$\boxed{\begin{aligned} x &= 2\tan\theta \\ dx &= 2\sec^2\theta\,d\theta \end{aligned}}$$

$\tan\theta = x/2 = \frac{opp}{adj}$

$$= 8\int \frac{\cancel{16}\tan^3\theta\cdot\sec^{\cancel{2}}\theta}{\sqrt{\cancel{4}\,\cancel{\sec^2\theta}}}\,d\theta$$



$$= 8\int \tan^3\theta\,\sec\theta\,d\theta$$

$\sec\theta = \frac{hyp}{adj}$

$\sec\theta = \frac{\sqrt{x^2+4}}{2}$

$$= 8\int \tan^2\theta\cdot\tan\theta\cdot\sec\theta\,d\theta$$

$$= 8\int (\sec^2\theta-1)\cdot\sec\theta\tan\theta\,d\theta$$

u-sub Box

$$\boxed{\begin{aligned} u &= \sec\theta \\ du &= \sec\theta\tan\theta\,d\theta \end{aligned}}$$

$$= 8\int u^2-1\,du = \frac{8}{3}u^3 - 8u + C$$

$$= \frac{8}{3}\sec^3\theta - 8\sec\theta + C$$

$$= \frac{\overset{1}{\cancel{8}}}{3}\,\frac{(x^2+4)^{3/2}}{\cancel{8}} - \frac{\overset{4}{\cancel{8}}\sqrt{x^2+4}}{\cancel{2}} + C$$

$$= \boxed{\frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C}$$

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