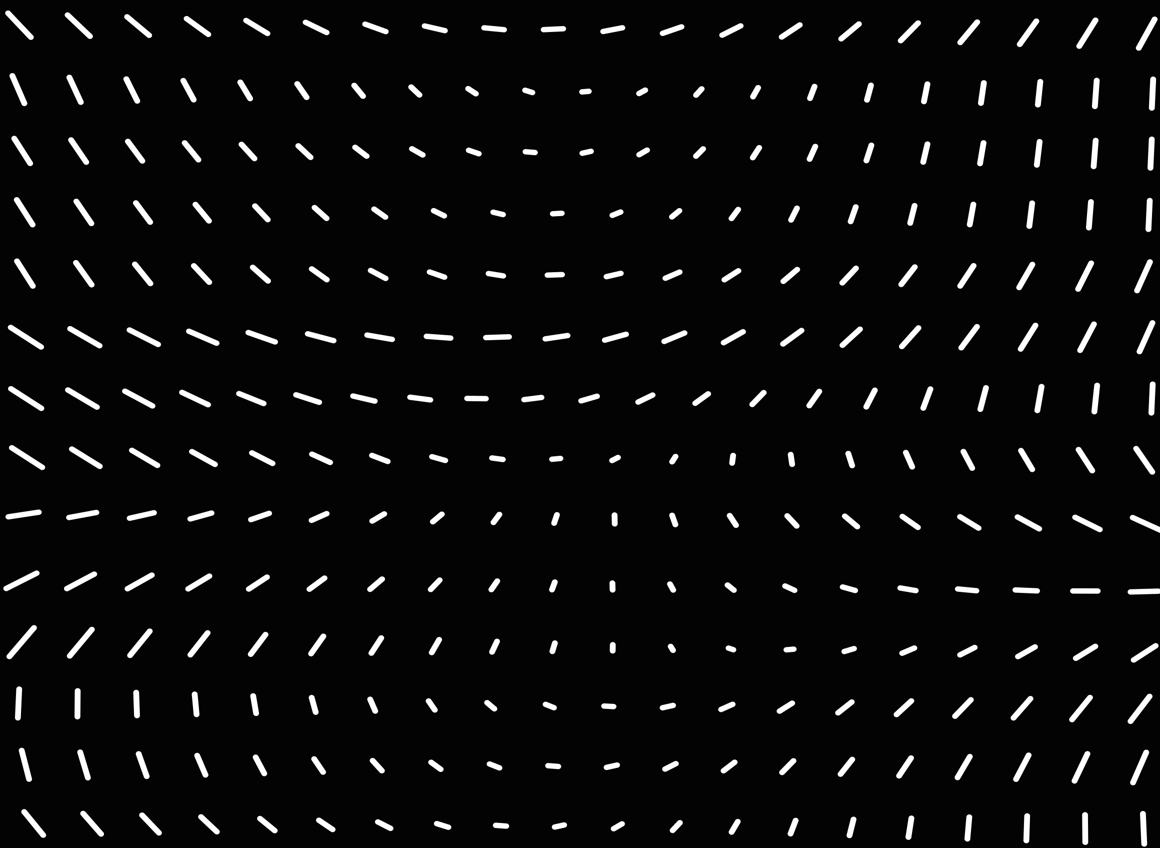

Math 1552: Sections 10.3, 10.4, 10.5

Convergence Tests
for Infinite Series

7	Jun 26 Section 10.4: Comparison Tests	Jun 27 WS 10.2 WS 10.3	Jun 28 Section 10.5: Ratio and Root Tests Review for Test 2	Jun 29 **Test #2 (8.4-8.5, 4.5, 8.8, 10.1-10.3)**	Jun 30 Section 10.5: cont. Section 10.6: Alternating Series
8	Jul 3 **NO CLASS** Independence Day	Jul 4 **NO CLASS** Student Recess	Jul 5 Section 10.6: cont. Section 10.7: Power series	Jul 6 WS 10.4 WS 10.5 Quiz #5 (10.4-10.5)	Jul 7 Section 10.7, cont.
9	Jul 10 Sections 10.8-10.9: Taylor polynomials and series	Jul 11 WS 10.6 WS 10.7	Jul 12 Sections 10.8-10.9, cont.	Jul 13 WS 10.8-10.9 Quiz #6 (10.6-10.8)	Jul 14 Sections 10.8-10.9, cont.
10	Jul 17 Sections 10.8-10.9, cont.	Jul 18 WS 10.8-10.9 (3 versions)	Jul 19 Sections 10.8-10.9, cont.	Jul 20 **Test #3 (10.4-10.9)**	Jul 21 Section 6.1: Volumes by Disks

THEOREM 10—Direct Comparison Test

Let $\sum a_n$ and $\sum b_n$ be two series with $0 \le a_n \le b_n$ for all n. Then

1. If $\sum b_n$ converges, then $\sum a_n$ also converges.
2. If $\sum a_n$ diverges, then $\sum b_n$ also diverges.

THEOREM 11—Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \ge N$ (N an integer).

1. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ and $c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.
2. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
3. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

$$\sum_{k=1}^{\infty} \frac{1}{1+2^k}$$

$$\sum_{k=2}^{\infty} \frac{1}{\sqrt{k}-1}$$

$$\sum_{n=1}^{\infty} \frac{5}{5n - 1}$$

THEOREM 11—Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

1. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ and $c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.
2. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
3. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

$$\sum_{n=1}^{\infty} \frac{2n + 1}{n^2 + 2n + 1}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$$

Section 10.4: 5, 9, 17, 21, 25, 34, 41
(extra practice: 13, 18, 23, 31, 39, 51, 58)

EXERCISES 10.4

Direct Comparison Test

In Exercises 1–8, use the Direct Comparison Test to determine if each series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2+30}$
2. $\sum_{n=1}^{\infty} \frac{n-1}{n^4+2}$
3. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$
4. $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$
5. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$
6. $\sum_{n=1}^{\infty} \frac{1}{n3^n}$
7. $\sum_{n=1}^{\infty} \sqrt{\frac{n+4}{n^4+4}}$
8. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}}$

Limit Comparison Test

In Exercises 9–16, use the Limit Comparison Test to determine if each series converges or diverges.

9. $\sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+3}$
(*Hint:* Limit Comparison with $\sum_{n=1}^{\infty}(1/n^2)$)
10. $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$
(*Hint:* Limit Comparison with $\sum_{n=1}^{\infty}(1/\sqrt{n})$)
11. $\sum_{n=2}^{\infty} \frac{n(n+1)}{(n^2+1)(n-1)}$
12. $\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$
13. $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}\,4^n}$
14. $\sum_{n=1}^{\infty} \left(\frac{2n+3}{5n+4}\right)^n$
15. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$
(*Hint:* Limit Comparison with $\sum_{n=2}^{\infty}(1/n)$)
16. $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right)$
(*Hint:* Limit Comparison with $\sum_{n=1}^{\infty}(1/n^2)$)

Determining Convergence or Divergence

Which of the series in Exercises 17–56 converge, and which diverge? Use any method, and give reasons for your answers.

17. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$
18. $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$
19. $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$
20. $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$
21. $\sum_{n=1}^{\infty} \frac{2n}{3n-1}$
22. $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$
23. $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$
24. $\sum_{n=3}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$
25. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$
26. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$
27. $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$
28. $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$
29. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln n}$
30. $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$
31. $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$
32. $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$
33. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$
34. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$
35. $\sum_{n=1}^{\infty} \frac{1-n}{n2^n}$
36. $\sum_{n=1}^{\infty} \frac{n+2^n}{n^2 2^n}$
37. $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$
38. $\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$
39. $\sum_{n=1}^{\infty} \frac{n+1}{n^2+3n}\cdot\frac{1}{5n}$
40. $\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$
41. $\sum_{n=1}^{\infty} \frac{2^n-n}{n2^n}$
42. $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}\,e^n}$
43. $\sum_{n=2}^{\infty} \frac{1}{n!}$
(*Hint:* First show that $(1/n!) \le (1/n(n-1))$ for $n \ge 2$.)
44. $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+2)!}$
45. $\sum_{n=1}^{\infty} \sin\frac{1}{n}$
46. $\sum_{n=1}^{\infty} \tan\frac{1}{n}$
47. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$
48. $\sum_{n=1}^{\infty} \frac{\sec^{-1} n}{n^{1.3}}$
49. $\sum_{n=1}^{\infty} \frac{\coth n}{n^2}$
50. $\sum_{n=1}^{\infty} \frac{\tanh n}{n^2}$
51. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$
52. $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$
53. $\sum_{n=1}^{\infty} \frac{1}{1+2+3+\cdots+n}$
54. $\sum_{n=1}^{\infty} \frac{1}{1+2^2+3^2+\cdots+n^2}$
55. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^2}$
56. $\sum_{n=2}^{\infty} \frac{(\ln n)^2}{n}$

Theory and Examples

57. Prove **(a)** Part 2 and **(b)** Part 3 of the Limit Comparison Test.
58. If $\sum_{n=1}^{\infty} a_n$ is a convergent series of nonnegative numbers, can anything be said about $\sum_{n=1}^{\infty}(a_n/n)$? Explain.
59. Suppose that $a_n > 0$ and $b_n > 0$ for $n \ge N$ (N an integer). If $\lim_{n\to\infty}(a_n/b_n) = \infty$ and $\sum a_n$ converges, can anything be said about $\sum b_n$? Give reasons for your answer.
60. Prove that if $\sum a_n$ is a convergent series of nonnegative terms, then $\sum a_n^2$ converges.
61. Suppose that $a_n > 0$ and $\lim_{n\to\infty} a_n = \infty$. Prove that $\sum a_n$ diverges.
62. Suppose that $a_n > 0$ and $\lim_{n\to\infty} n^2 a_n = 0$. Prove that $\sum a_n$ converges.

10.5 Absolute Convergence; The Ratio and Root Tests

7	Jun 26 Section 10.4: Comparison Tests	Jun 27 WS 10.2 WS 10.3	Jun 28 Section 10.5: Ratio and Root Tests Review for Test 2	Jun 29 **Test #2 (8.4-8.5, 4.5, 8.8, 10.1-10.3)**	Jun 30 Section 10.5: cont. Section 10.6: Alternating Series
8	Jul 3 **NO CLASS** Independence Day	Jul 4 **NO CLASS** Student Recess	Jul 5 Section 10.6: cont. Section 10.7: Power series	Jul 6 WS 10.4 WS 10.5 Quiz #5 (10.4-10.5)	Jul 7 Section 10.7, cont.
9	Jul 10 Sections 10.8-10.9: Taylor polynomials and series	Jul 11 WS 10.6 WS 10.7	Jul 12 Sections 10.8-10.9, cont.	Jul 13 WS 10.8-10.9 Quiz #6 (10.6-10.8)	Jul 14 Sections 10.8-10.9, cont.
10	Jul 17 Sections 10.8-10.9, cont.	Jul 18 WS 10.8-10.9 (3 versions)	Jul 19 Sections 10.8-10.9, cont.	Jul 20 **Test #3 (10.4-10.9)**	Jul 21 Section 6.1: Volumes by Disks

Recap of last class:

- *Divergence test*: if the limit is not 0, the series diverges
- *Comparison test*: find a bigger series that converges or a smaller series that diverges
- *Integral test*: use with a function that has an "easy" antiderivative

Ratio Test

Let $\sum_{k=1}^{\infty} a_k$ be a series with all positive terms.

Let $L = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$.

(a) If $L < 1$, then $\sum_{k=1}^{\infty} a_k$ coverges.

(b) If $L > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.

(c) If $L = 1$, then the test is *INCONCLUSIVE*!!!

$$\sum_{k=1}^{\infty} \frac{3^k}{k^2}$$

$$\sum_{k=1}^{\infty} \frac{k \cdot 3^k}{(2k)!}$$

Root Test

Let $\sum_{k=1}^{\infty} a_k$ be a series with all positive terms.

Let $R = \lim_{k\to\infty} \sqrt[k]{a_k}$.

(a) If $R < 1$, then $\sum_{k=1}^{\infty} a_k$ coverges.

(b) If $R > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.

(c) If $R = 1$, then the test is *INCONCLUSIVE*!!!

$$\sum_{k=1}^{\infty} \left(1 - \frac{2}{k}\right)^{k^2}$$

Recap of last class:

- *Limit Comparison test*: pick a series that you know converges or diverges.

If the limit of the ratio of terms in your series to the given series approaches a finite, positive number, then both series either converge or diverge.

Topics on Exam 2

* trig sub
* partial fractions
* L'Hop
* Improper integrals
* sequences
* Convergence tests for series
  - ↳ integral test
  - ↳ geometric series
  - ↳ telescoping series
* Divergence test

Ex.

$$\sum_{k=1}^{\infty} \frac{3^k}{k^2}$$

Ex.

$$\sum_{k=1}^{\infty} \frac{k \cdot 3^k}{(2k)!}$$

Ex.

$$\sum_{k=1}^{\infty}\left(1+\frac{2}{k}\right)^{k^2}$$

Ex.

$$\sum_{n=1}^{\infty}\frac{4^n}{(3n)^n}$$

Ex.

$$\sum_{n=1}^{\infty}\left(\frac{4n+1}{3n+1}\right)^n$$

Section 10.5: 7, 13, 15, 17, 18, 31, 34, 57, 63, 67
(extra practice: 7, 17, 18, 19, 21, 25, 37, 42, 59)

EXERCISES 10.5

Using the Ratio Test

In Exercises 1–8, use the Ratio Test to determine if each series converges absolutely or diverges.

1. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$
2. $\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{3^n}$
3. $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$
4. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$
5. $\sum_{n=1}^{\infty} \frac{n^4}{(-4)^n}$
6. $\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$
7. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2(n+2)!}{n!\, 3^{2n}}$
8. $\sum_{n=1}^{\infty} \frac{n5^n}{(2n+3)\ln(n+1)}$

Using the Root Test

In Exercises 9–16, use the Root Test to determine if each series converges absolutely or diverges.

9. $\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n}$
10. $\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$
11. $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$
12. $\sum_{n=1}^{\infty} \left(-\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$
13. $\sum_{n=1}^{\infty} \frac{-8}{(3+(1/n))^{2n}}$
14. $\sum_{n=1}^{\infty} \sin^n\left(\frac{1}{\sqrt{n}}\right)$
15. $\sum_{n=1}^{\infty} (-1)^n \left(1 - \frac{1}{n}\right)^{n^2}$

    (*Hint:* $\lim_{n\to\infty} (1 + x/n)^n = e^x$)
16. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{1+n}}$

Determining Convergence or Divergence

In Exercises 17–46, use any method to determine if the series converges or diverges. Give reasons for your answer.

17. $\sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}$
18. $\sum_{n=1}^{\infty} (-1)^n n^2 e^{-n}$
19. $\sum_{n=1}^{\infty} n!(-e)^{-n}$
20. $\sum_{n=1}^{\infty} \frac{n!}{10^n}$
21. $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$
22. $\sum_{n=1}^{\infty} \left(\frac{n-2}{n}\right)^n$
23. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.25^n}$
24. $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n}$
25. $\sum_{n=1}^{\infty} (-1)^n \left(1 - \frac{3}{n}\right)^n$
26. $\sum_{n=1}^{\infty} \left(1 - \frac{1}{3n}\right)^n$
27. $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$
28. $\sum_{n=1}^{\infty} \frac{(-\ln n)^n}{n^n}$
29. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$
30. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$
31. $\sum_{n=1}^{\infty} \frac{e^n}{n^e}$
32. $\sum_{n=1}^{\infty} \frac{n \ln n}{(-2)^n}$
33. $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$
34. $\sum_{n=1}^{\infty} e^{-n}(n^3)$
35. $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$
36. $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$
37. $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$
38. $\sum_{n=1}^{\infty} \frac{n!}{(-n)^n}$
39. $\sum_{n=2}^{\infty} \frac{-n}{(\ln n)^n}$
40. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^{(n/2)}}$
41. $\sum_{n=1}^{\infty} \frac{n! \ln n}{n(n+2)!}$
42. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3 2^n}$
43. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$
44. $\sum_{n=1}^{\infty} \frac{(2n+3)(2^n+3)}{3^n+2}$
45. $\sum_{n=3}^{\infty} \frac{2^n}{n^2}$
46. $\sum_{n=3}^{\infty} \frac{2^{n^2}}{n^{2^n}}$

55. $a_1 = \frac{1}{3}, \quad a_{n+1} = \sqrt[n]{a_n}$
56. $a_1 = \frac{1}{2}, \quad a_{n+1} = (a_n)^{n+1}$

Convergence or Divergence

Which of the series in Exercises 57–64 converge, and which diverge? Give reasons for your answers.

57. $\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$
58. $\sum_{n=1}^{\infty} \frac{(-1)^n (3n)!}{n!(n+1)!(n+2)!}$
59. $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$
60. $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^n}{n^{(n^2)}}$
61. $\sum_{n=1}^{\infty} \frac{n^n}{2^{(n^2)}}$
62. $\sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$
63. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{4^n 2^n n!}$
64. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{[2 \cdot 4 \cdot \cdots \cdot (2n)](3^n+1)}$
65. Assume that b_n is a sequence of positive numbers converging to $4/5$. Determine if the following series converge or diverge.

    a. $\sum_{n=1}^{\infty} (b_n)^{1/n}$
    b. $\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n (b_n)$
    c. $\sum_{n=1}^{\infty} (b_n)^n$
    d. $\sum_{n=1}^{\infty} \frac{1000^n}{n! + b_n}$
66. Assume that b_n is a sequence of positive numbers converging to $1/3$. Determine if the following series converge or diverge.

    a. $\sum_{n=1}^{\infty} \frac{b_{n+1}b_n}{n4^n}$
    b. $\sum_{n=1}^{\infty} \frac{n^n}{n!\, b_1^2 b_2^2 \cdots b_n^2}$

Theory and Examples

67. Neither the Ratio Test nor the Root Test helps with p-series. Try them on

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

and show that both tests fail to provide information about convergence.

Math 1552
Section 10.6

Alternating Series

7	Jun 26 Section 10.4: Comparison Tests	Jun 27 WS 10.2 WS 10.3	Jun 28 Section 10.5: Ratio and Root Tests Review for Test 2	Jun 29 **Test #2 (8.4-8.5, 4.5, 8.8, 10.1-10.3)**	Jun 30 Section 10.5: cont. Section 10.6: Alternating Series
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Alternating Series Test

Let $\sum_k a_k$ be an alternating series.

(a) If $\sum_k |a_k|$ converges, then the series *converges absolutely*.

(b) If (a) fails, then if :

i) $\{|a_n|\}$ is a decreasing sequence, and

ii) $\lim_{n\to\infty} |a_n| = 0$,

then the series *converges conditionally*.

(c) Otherwise, the series *diverges*.

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k+4}}$$

$$\sum_{k=1}^{\infty} (-1)^k \frac{k}{3^k}$$

$$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{k^3+2k+1}$$

Example:

Estimate the sum of the series below within an error range of 0.001.

$$\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)!}$$

Estimating the Sum

Let $\sum_k a_k$ be a convergent alternating series with a sum of L.

Then : $|s_n - L| < |a_{n+1}|$.

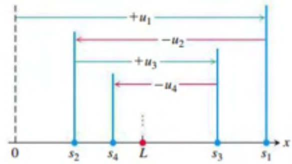
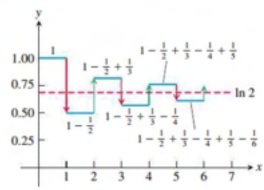


FIGURE 10.15 The partial sums of an alternating series that satisfies the hypotheses of Theorem 15 for $N = 1$ straddle the limit from the beginning.

EXAMPLE 1 The alternating harmonic series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$



(b)

Section 10.6: 6, 19, 21, 23, 25, 27, 34, 35, 37, 47

EXERCISES 10.6

(extra practice: 20, 22, 27, 31, 91)

Determining Convergence or Divergence

In Exercises 1–14, determine if the alternating series converges or diverges. Some of the series do not satisfy the conditions of the Alternating Series Test.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{3/2}}$

3. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n3^n}$

4. $\sum_{n=2}^{\infty}(-1)^{n}\frac{4}{(\ln n)^2}$

5. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n^2+1}$

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+5}{n^2+4}$

7. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n^2}$

8. $\sum_{n=1}^{\infty}(-1)^{n}\frac{10^n}{(n+1)!}$

9. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{10}\right)^n$

10. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\ln n}$

11. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$

12. $\sum_{n=1}^{\infty}(-1)^{n}\ln\left(1+\frac{1}{n}\right)$

13. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n}+1}{n+1}$

14. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3\sqrt{n+1}}{\sqrt{n}+1}$

Absolute and Conditional Convergence

Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

15. $\sum_{n=1}^{\infty}(-1)^{n+1}(0.1)^n$

16. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

17. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\sqrt{n}}$

18. $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+\sqrt{n}}$

19. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$

20. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{2^n}$

21. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n+3}$

22. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\sin n}{n^2}$

23. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n}$

24. $\sum_{n=1}^{\infty}\frac{(-2)^{n+1}}{n+5^n}$

25. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$

26. $\sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt[n]{10})$

27. $\sum_{n=1}^{\infty}(-1)^{n}n^2(2/3)^n$

28. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\ln n}$

29. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\tan^{-1}n}{n^2+1}$

30. $\sum_{n=1}^{\infty}(-1)^{n}\frac{\ln n}{n-\ln n}$

31. $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n+1}$

32. $\sum_{n=1}^{\infty}(-5)^{-n}$

33. $\sum_{n=1}^{\infty}\frac{(-100)^n}{n!}$

34. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2+2n+1}$

35. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n\sqrt{n}}$

36. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}$

37. $\sum_{n=1}^{\infty}\frac{(-1)^n(n+1)^n}{(2n)^n}$

38. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(n!)^2}{(2n)!}$

Error Estimation

In Exercises 49–52, estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

49. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

50. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{10^n}$

51. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.01)^n}{n}$ As you will see in Section 10.7, the sum is ln (1.01).

52. $\frac{1}{1+t}=\sum_{n=0}^{\infty}(-1)^n t^n,\quad 0<t<1$

In Exercises 53–56, determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

53. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n^2+3}$

54. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

55. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(n+3\sqrt{n})^3}$

56. $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{\ln(\ln(n+2))}$

In Exercises 57–82, use any method to determine whether the series converges or diverges. Give reasons for your answer.

57. $\sum_{n=1}^{\infty}\frac{3^n}{n^n}$

58. $\sum_{n=1}^{\infty}\frac{3^n}{n^3}$

59. $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$

60. $\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right)$

61. $\sum_{n=0}^{\infty}(-1)^{n}\frac{(n+2)!}{(2n)!}$

62. $\sum_{n=2}^{\infty}\frac{(3n)!}{(n!)^3}$

63. $\sum_{n=1}^{\infty}n^{-2/\sqrt{5}}$

64. $\sum_{n=2}^{\infty}\frac{3}{10+n^{4/3}}$

65. $\sum_{n=1}^{\infty}\left(1-\frac{2}{n}\right)^{n^2}$

66. $\sum_{n=0}^{\infty}\left(\frac{n+1}{n+2}\right)^n$

67. $\sum_{n=1}^{\infty}\frac{n-2}{n^2+3n}\left(-\frac{2}{3}\right)^n$

68. $\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}\left(\frac{3}{2}\right)^n$

39. $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{2^n n! n}$

40. $\sum_{n=1}^{\infty}(-1)^n \frac{(n!)^2 3^n}{(2n+1)!}$

41. $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$

42. $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n^2+n}-n\right)$

43. $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$

44. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$

45. $\sum_{n=1}^{\infty}(-1)^n \operatorname{sech} n$

46. $\sum_{n=1}^{\infty}(-1)^n \operatorname{csch} n$

47. $\frac{1}{4}-\frac{1}{6}+\frac{1}{8}-\frac{1}{10}+\frac{1}{12}-\frac{1}{14}+\cdots$

48. $1+\frac{1}{4}-\frac{1}{9}-\frac{1}{16}+\frac{1}{25}+\frac{1}{36}-\frac{1}{49}-\frac{1}{64}+\cdots$

69. $\frac{1}{2}-\frac{1}{2}+\frac{1}{2}-\frac{1}{2}+\frac{1}{2}-\frac{1}{2}+\cdots$

70. $1-\frac{1}{8}+\frac{1}{64}-\frac{1}{512}+\frac{1}{4096}-\cdots$

71. $\sum_{n=3}^{\infty}\sin\left(\frac{1}{\sqrt{n}}\right)$

72. $\sum_{n=1}^{\infty}\tan(n^{1/n})$

73. $\sum_{n=2}^{\infty}\frac{n}{\ln n}$

74. $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$

75. $\sum_{n=2}^{\infty}\ln\left(\frac{n+2}{n+1}\right)$

76. $\sum_{n=2}^{\infty}\left(\frac{\ln n}{n}\right)^3$

77. $\sum_{n=2}^{\infty}\frac{1}{1+2+2^2+\cdots+2^n}$

78. $\sum_{n=2}^{\infty}\frac{1+3+3^2+\cdots+3^{n-1}}{1+2+3+\cdots+n}$

79. $\sum_{n=0}^{\infty}(-1)^n \frac{e^n}{e^n+e^{n^2}}$

80. $\sum_{n=0}^{\infty}\frac{(2n+3)(2^n+3)}{3^n+2}$

81. $\sum_{n=1}^{\infty}\frac{n^2 3^n}{3\cdot 5\cdot 7\cdots(2n+1)}$

82. $\sum_{n=1}^{\infty}\frac{4\cdot 6\cdot 8\cdots(2n)}{5^{n+1}(n+2)!}$

T Approximate the sums in Exercises 83 and 84 with an error of magnitude less than 5×10^{-6}.

83. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n)!}$ As you will see in Section 10.9, the sum is cos 1, the cosine of 1 radian.

84. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!}$ As you will see in Section 10.9 the sum is e^{-1}.

Theory and Examples

85. a. The series

$$\frac{1}{3}-\frac{1}{2}+\frac{1}{9}-\frac{1}{4}+\frac{1}{27}-\frac{1}{8}+\cdots+\frac{1}{3^n}-\frac{1}{2^n}+\cdots$$

does not meet one of the conditions of Theorem 14. Which one?

b. Use Theorem 17 to find the sum of the series in part (a).

90. Show that if $\sum_{n=1}^{\infty} a_n$ converges absolutely, then

$$\left|\sum_{n=1}^{\infty} a_n\right| \le \sum_{n=1}^{\infty} |a_n|.$$

91. Show that if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge absolutely, then so do the following.

a. $\sum_{n=1}^{\infty}(a_n+b_n)$

b. $\sum_{n=1}^{\infty}(a_n-b_n)$

c. $\sum_{n=1}^{\infty} ka_n$ (k any number)

92. Show by example that $\sum_{n=1}^{\infty} a_n b_n$ may diverge even if $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge.

93. If $\sum a_n$ converges absolutely, prove that $\sum a_n^2$ converges.

94. Does the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)$$

converge or diverge? Justify your answer.

T 95. In the alternating harmonic series, suppose the goal is to arrange the terms to get a new series that converges to $-1/2$. Start the new