WEEK 11

Volumes of Revolution

Disks/Washers & Cylindrical Shells

Math 1552

Sections 6.1-6.2:

Volumes of Revolution

8	Jul 3 **NO CLASS** Independence Day	Jul 4 **NO CLASS** Student Recess	Jul 5 Section 10.6: cont. Section 10.7: Power series	Jul 6 WS 10.4 WS 10.5 Quiz #5 (10.4-10.5)	Jul 7 Section 10.7, cont.
9	Jul 10 Sections 10.8-10.9: Taylor polynomials and series	Jul 11 WS 10.6 WS 10.7	Jul 12 Sections 10.8-10.9, cont.	Jul 13 WS 10.8-10.9 Quiz #6 (10.6-10.8)	Jul 14 Sections 10.8-10.9, cont.
10	Jul 17 Sections 10.8-10.9, cont.	Jul 18 WS 10.8-10.9 (3 versions)	Jul 19 Sections 10.8-10.9, cont.	Jul 20 **Test #3 (10.4-10.9)**	Jul 21 Section 6.1: Volumes by Disks
11	Jul 24 Section 6.1: Volumes by Cylindrical Shells Final Review	Jul 25 WS 6.1-6.2 *Last day for MML homework*	Jul 26 Reading Day	Jul 27	Jul 28 **FINAL EXAM** **11:20 AM – 2:10 PM**
12	Jul 31	Aug 1	Aug 2	Aug 3	Aug 4

Learning Goals

- Set up and evaluate integrals using the disk method
- Set up and evaluate integrals using the method of cylindrical shells
- Apply the "washer" method to either method above
- Adjust the standard formulas to rotate a region around any horizontal or vertical line

Important Notes about Disks:

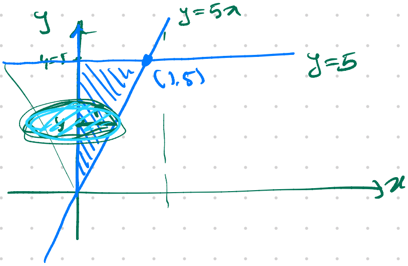
- The variable of integration *always* matches the axis of revolution.
- If you revolve about a line other than the x- or y-axis, you will need to adjust the formula to find the new radius.
- If you revolve a region bounded by two curves, you will need to apply the *washer method*.

Example 1:

Find the volume of the solid generated by revolving the region bounded by $y=5x$, $x=0$, and $y=5$ about the y-axis.

know y want x

$y=5x \Rightarrow x=\frac{y}{5}$



Formula

$$\boxed{\int_a^b A(y)\,dy}$$

Need

$a=?$

$b=?$

$A(y)=?$

$A(y)=\pi r^2$ Disks

& $r=\frac{y}{5}$

CLOs

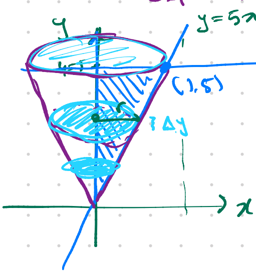
* Canvas
* Piazza
* Lecture
* Tell TAs to tell Students
* Lecture Notes
* T-shirt

$$\text{Volume}=\int_0^5 \pi\left(\frac{y}{5}\right)^2 dy=\pi\int_0^5 \frac{y^2}{25}\,dy$$

$$=\frac{\pi}{25}\cdot\frac{y^3}{3}\Big|_0^5=\frac{125}{25}\cdot\frac{\pi}{3}-0=\boxed{\frac{5\pi}{3}}$$

Power ↓ Motivating Stuff

Q: Volume of this cone?



From Exam 1

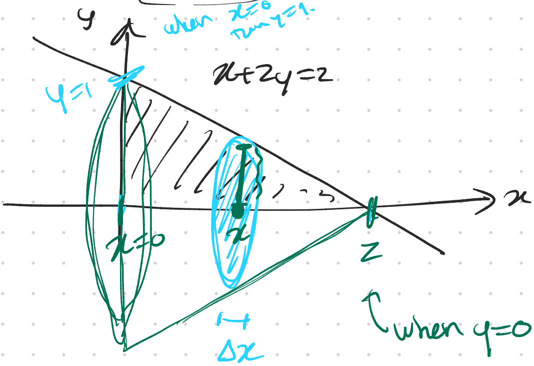
$$\int_a^b f(x)\,dx=\lim_{N\to\infty}\sum_{i=1}^{N} f(x_i)\Delta x$$

$A(y)=\pi r^2$

$$\boxed{\text{Volume}}\approx\sum_{i=1}^{N} A(y)\cdot\Delta y$$

$$=\lim_{N\to\infty}\sum_{i=1}^{N} A(y_i)\,\Delta y=\boxed{\int_a^b A(y)\,dy}$$

Ex. Find the volume of the solid obtained by revolving the shape below about the x-axis (Section 6.1 #17)



$$\text{Volume} = \int_a^b A(x)\,dx$$

$$= \int_0^2 \pi\left(\frac{2-x}{2}\right)^2 dx$$

radius?

$x+2y=2$

Solve for y $\Rightarrow 2y = 2-x$

$\Rightarrow y = \frac{2-x}{2}$

$$= \pi\int_0^2 \frac{1}{4}\left(x^2-4x+4\right)dx$$

Simplify FIRST THEN INTEGRATE

$$= \frac{\pi}{4}\int_0^2 x^2-4x+4\,dx$$

$$= \frac{\pi}{4}\left(\frac{1}{3}x^3 - 2x^2 + 4x \Big|_0^2\right)$$

$$= \frac{\pi}{4}\left[\left(\frac{8}{3} - 8 + 8\right) - 0\right]$$

$$= \frac{\pi}{4}\cdot\frac{8}{3} = \boxed{\frac{2\pi}{3}}$$

Example 2:

Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{1-x^2}$ and $x+y=1$ about the x-axis.

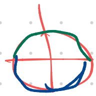
$y=1-x$

Without the picture

$x^2+y^2=1$

$\Rightarrow y^2=1-x^2$

$\Rightarrow y=\pm\sqrt{1-x^2}$



Step 1: Find x-intercepts

(The x-values of the intersection points.)

Set $y=y$ & solve for x.

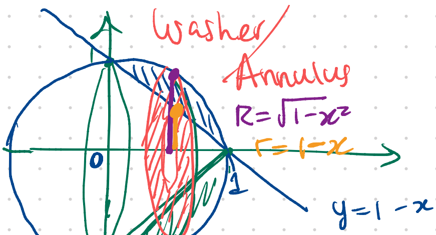
$$\sqrt{1-x^2}=1-x$$

$$\Rightarrow \quad 1-x^2=(1-x)^2$$

$$\Rightarrow \quad 1-x^2=x^2-2x+1$$

$$\Rightarrow \quad 0=2x^2-2x=2x(x-1)$$

So $x=0,\ x=1$



$$A(x)=\pi R^2-\pi r^2 = \boxed{\pi(R^2-r^2)}$$

big circle area

little circle area

$$A(x)=\pi\left(\sqrt{1-x^2}\right)^2-\pi(1-x)^2$$

$$\text{Volume}=\int_0^1 A(x)\,dx$$

$$=\int_0^1 \pi\left(\sqrt{1-x^2}\right)^2-\pi(1-x)^2\,dx$$

$$=\pi\int_0^1 1-x^2-(x^2-2x+1)\,dx$$

$$=\pi\int_0^1 -2x^2+2x\,dx=\pi\left(-\frac{2}{3}x^3+x^2\Big|_0^1\right)=\pi\left(\left(-\frac{2}{3}+1\right)-0\right)$$

$$=\boxed{\pi/3}\ \text{units}^3$$

Example 3:

Find the volume of the solid generated by revolving the region bounded by:

$y=\sqrt{x+1}, x=3,$ and the x-axis about the line $y=-1$.

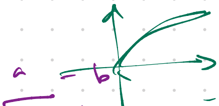
$$\boxed{\int_a^b A(x)\,dx}$$

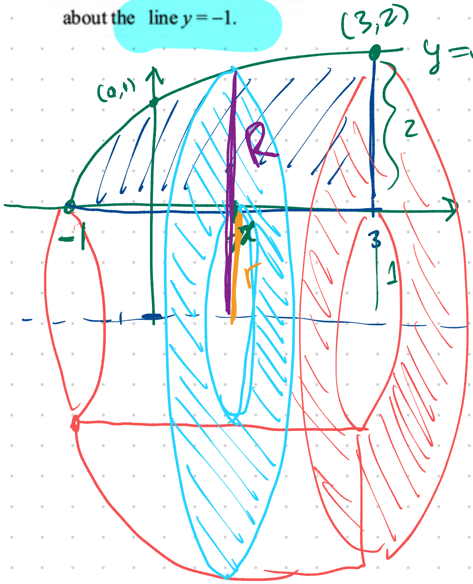
ALWAYS

total dist from a to b is $(a \geq b)$

$|a-b| = a-b$

$y=\sqrt{x}$ $\quad y^2=x$





$R=\sqrt{x+1}-(-1)$

$r=1$

$$A(x)=\pi\left(R^2-r^2\right)$$

$$=\pi\left(\left(\sqrt{x+1}+1\right)^2-1^2\right)$$

$$\int_{-1}^{3} A(x)\,dx$$

$$=\pi\int_{-1}^{3}\left(\sqrt{x+1}+1\right)^2-1^2\,dx$$

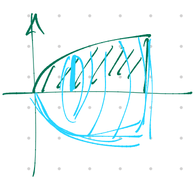
$$=\pi\int_{-1}^{3}\left(\sqrt{x+1}\right)^2+2\sqrt{x+1}\cancel{+1}\cancel{-1}\,dx$$

$$=\pi\int_{-1}^{3} x+1+2\sqrt{x+1}\,dx$$

$$\int\sqrt{x+1}\,dx = \frac{2}{3}(x+1)^{3/2}+C$$

$$=\pi\left[\frac{1}{2}x^2+x+\frac{4}{3}(x+1)^{3/2}\Big|_{-1}^{3}\right]$$

$$=\pi\left[\left(\frac{9}{2}+3+\frac{4}{3}4^{3/2}\right)-\left(\underbrace{\frac{1}{2}-1}_{-1/2}+0\right)\right]$$



$$=\pi\left[\frac{9}{2}+\frac{1}{2}+3+\frac{32}{3}\right]=\pi\left[8+\frac{32}{3}\right]=\boxed{\frac{56\pi}{3}}$$

Example: Set up the integral to find the volume bounded by

$y = x+2$ and $y = x^2$, $\boxed{x \ge 0,}$

about the x-axis.

(A) $V = \pi\int_{-1}^{2}\left[(x+2)^2-(x^2)^2\right]dx$

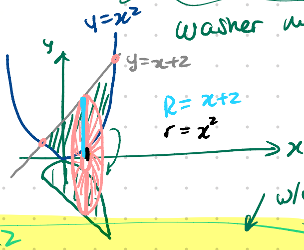
(B) $V = \pi\int_{0}^{2}\left[(x+2)^2-(x^2)^2\right]dx$

(C) $V = \pi\int_{-1}^{2}\left[(x^2)^2-(x+2)^2\right]dx$

(D) $V = \pi\int_{0}^{2}\left[(x^2)^2-(x+2)^2\right]dx$

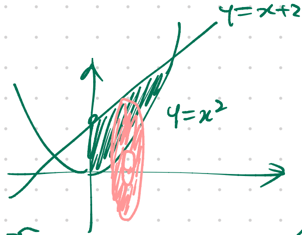
$$\text{Volume} = \int_a^b \pi\left(R^2 - r^2\right)dx$$

washer method.



$R = x+2$

$r = x^2$



w/o the restriction $x \ge 0$

$$\boxed{\text{Volume} = \int_{-1}^{2}\pi\left[(x+2)^2 - (x^2)^2\right]dx}$$

question as written

$$\boxed{\int_0^2 \pi\left[(x+2)^2 - (x^2)^2\right]dx}$$

To find a & b we have to set $y=y$ & solve for x

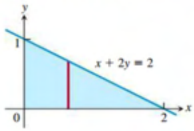
$$x+2 = x^2 \Rightarrow x^2 - x - 2 = 0$$
$$\Rightarrow (x-2)(x+1) = 0$$
$$\Rightarrow x = -1, 2$$

Section 6.1: 9, 17, 21, 23, 29, 31, 43, 47, 53, 55

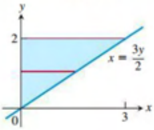
Volumes by the Disk Method

In Exercises 17–20, find the volume of the solid generated by revolving the shaded region about the given axis.

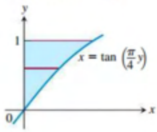
17. About the x-axis



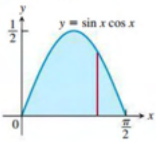
18. About the y-axis



19. About the y-axis



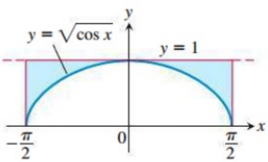
20. About the x-axis



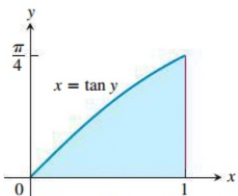
Volumes by the Washer Method

Find the volumes of the solids generated by revolving the shaded regions in Exercises 39 and 40 about the indicated axes.

39. The x-axis



40. The y-axis



Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 41–46 about the x-axis.

41. $y = x,\quad y = 1,\quad x = 0$

42. $y = 2\sqrt{x},\quad y = 2,\quad x = 0$

43. $y = x^2 + 1,\quad y = x + 3$

44. $y = 4 - x^2,\quad y = 2 - x$

45. $y = \sec x,\quad y = \sqrt{2},\quad -\pi/4 \le x \le \pi/4$

46. $y = \sec x,\quad y = \tan x,\quad x = 0,\quad x = 1$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 21–30 about the x-axis.

21. $y = x^2,\quad y = 0,\quad x = 2$ **22.** $y = x^3,\quad y = 0,\quad x = 2$

23. $y = \sqrt{9 - x^2},\quad y = 0$ **24.** $y = x - x^2,\quad y = 0$

25. $y = \sqrt{\cos x},\quad 0 \le x \le \pi/2,\quad y = 0,\quad x = 0$

26. $y = \sec x,\quad y = 0,\quad x = -\pi/4,\quad x = \pi/4$

27. $y = e^{-x},\quad y = 0,\quad x = 0,\quad x = 1$

28. The region between the curve $y = \sqrt{\cot x}$ and the x-axis from $x = \pi/6$ to $x = \pi/2$

29. The region between the curve $y = 1/(2\sqrt{x})$ and the x-axis from $x = 1/4$ to $x = 4$

30. $y = e^{x-1},\quad y = 0,\quad x = 1,\quad x = 3$

In Exercises 31 and 32, find the volume of the solid generated by revolving the region about the given line.

31. The region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec x \tan x$, and on the left by the y-axis, about the line $y = \sqrt{2}$

32. The region in the first quadrant bounded above by the line $y = 2$, below by the curve $y = 2 \sin x$, $0 \le x \le \pi/2$, and on the left by the y-axis, about the line $y = 2$

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves in Exercises 33–38 about the y-axis.

33. The region enclosed by $x = \sqrt{5}y^2,\quad x = 0,\quad y = -1,\quad y = 1$

34. The region enclosed by $x = y^{3/2},\quad x = 0,\quad y = 2$

35. The region enclosed by $x = \sqrt{2 \sin 2y},\quad 0 \le y \le \pi/2,\quad x = 0$

36. The region enclosed by $x = \sqrt{\cos(\pi y/4)},\quad -2 \le y \le 0,\quad x = 0$

37. $x = 2/\sqrt{y + 1},\quad x = 0,\quad y = 0,\quad y = 3$

38. $x = \sqrt{2y}/(y^2 + 1),\quad x = 0,\quad y = 1$

In Exercises 47–50, find the volume of the solid generated by revolving each region about the y-axis.

47. The region enclosed by the triangle with vertices (1, 0), (2, 1), and (1, 1)

48. The region enclosed by the triangle with vertices (0, 1), (1, 0), and (1, 1)

49. The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$

50. The region in the first quadrant bounded on the left by the circle $x^2 + y^2 = 3$, on the right by the line $x = \sqrt{3}$, and above by the line $y = \sqrt{3}$

In Exercises 51 and 52, find the volume of the solid generated by revolving each region about the given axis.

51. The region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, about the line $x = -1$

52. The region in the second quadrant bounded above by the curve $y = -x^3$, below by the x-axis, and on the left by the line $x = -1$, about the line $x = -2$

Volumes of Solids of Revolution

53. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about

a. the x-axis. **b.** the y-axis.

c. the line $y = 2$. **d.** the line $x = 4$.

54. Find the volume of the solid generated by revolving the triangular region bounded by the lines $y = 2x$, $y = 0$, and $x = 1$ about

a. the line $x = 1$. **b.** the line $x = 2$.

55. Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about

a. the line $y = 1$. **b.** the line $y = 2$.

c. the line $y = -1$.

56. By integration, find the volume of the solid generated by revolving the triangular region with vertices (0, 0), $(b, 0)$, $(0, h)$ about

a. the x-axis. **b.** the y-axis.

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Notes about Shell Method:

- In the shell method, the variable of integration is the *opposite* of the axis of revolution.
- To use the washer method with shells:

$$V = 2\pi\int_a^b x[f(x)-g(x)]dx = 2\pi\int_a^b x[top - bottom]dx$$

OR

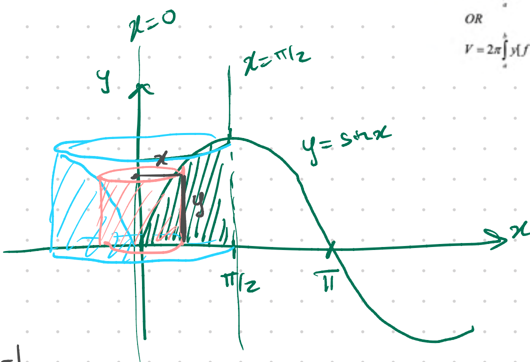
$$V = 2\pi\int_a^b y[f(y)-g(y)]dy = 2\pi\int_a^b y[right - left]dy$$

Example 4:

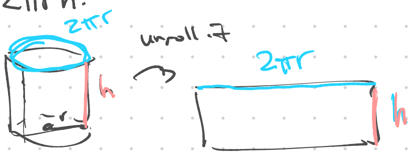
Find the volume of the solid generated by revolving the region bounded by the curves:

$y = \sin x$, the x-axis, and the lines

$x = 0,\ x = \dfrac{\pi}{2}$ about the y-axis.



$A = 2\pi r h.$



$$\text{Volume} = \int_a^b A(x)\,dx = \int_0^{\pi/2} 2\pi \cdot r \cdot h\,dx = \int_0^{\pi/2} 2\pi x \cdot \sin x\,dx$$

IBP

$$\boxed{\begin{array}{ll} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}}$$

$$= 2\pi\left(-x\cos x\Big|_0^{\pi/2} - \int_0^{\pi/2} -\cos x\,dx\right)$$

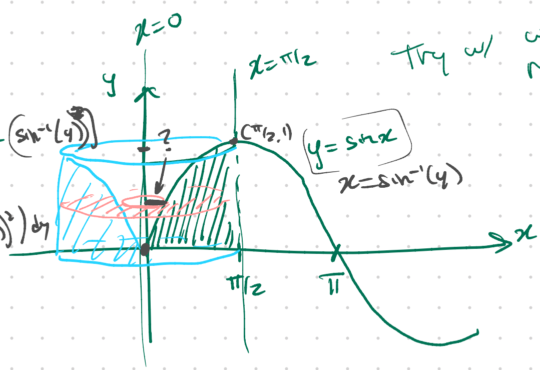
$$= 2\pi\left(\left(-\frac{\pi}{2}0 - 0\cdot 1\right) + \sin x\Big|_0^{\pi/2}\right)$$

$$= 2\pi(1-0) = \boxed{2\pi}$$

Try w/ washer method?

$$A(y) = \pi\left(\left(\frac{\pi}{2}\right)^2 - \left(\sin^{-1}(y)\right)^2\right)$$

$$\int_0^1 \pi\left(\left(\frac{\pi}{2}\right)^2 - \left(\sin^{-1}(y)\right)^2\right) dy$$



Example: Set up the integral to find the volume bounded by

$y = x + 2$ and $y = x^2$

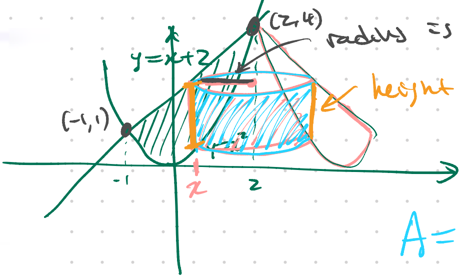
about the line $x = 2$.

(A) $V = 2\pi\int_1^4 (y+2)\left[(y-2)-\sqrt{y}\right]dy$

(B) $V = 2\pi\int_{-1}^2 (x-2)\left[(x+2)-x^2\right]dx$

(C) $V = 2\pi\int_1^4 (2-y)\left[(y-2)-\sqrt{y}\right]dy$

(D) $V = 2\pi\int_{-1}^2 (2-x)\left[(x+2)-x^2\right]dx$



radius is $2-x$

height is $(x+2)-x^2$

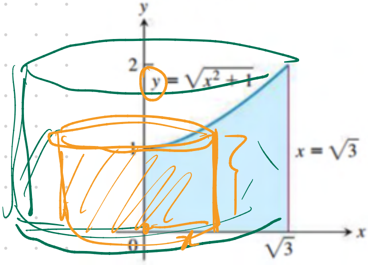
$A = 2\pi r h$

$$\text{Volume} = \int_{-1}^{2} A(x)\,dx$$

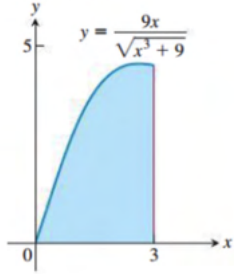
$$= \int_{-1}^{2} 2\pi(2-x)\cdot\left[(x+2)-x^2\right]dx$$

= mult. out & use power rule.

5. The y-axis



6. The y-axis



#5. $\text{Vol} = \int_a^b A(x)\,dx$ & $A(x) = 2\pi\, r(x)\cdot h(x)$ [Shells]

$\text{Vol} = \int_a^b A(y)\,dy$ & $A(y) = \pi\left[(R(y))^2 - (r(y))^2\right]$ (washers)

$h = \sqrt{x^2+1}$

$r = x$

$A = 2\pi r h$

$$\text{Vol} = \int_0^{\sqrt{3}} 2\pi x \cdot \sqrt{x^2+1}\,dx$$

$\sqrt{u} = u^{1/2}$

u-sub BOX

$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \end{aligned}}$$

$$= \int_1^4 \pi\sqrt{u}\,du$$

$$= \pi \cdot \frac{2}{3}u^{3/2}\Big|_1^4 = \pi\left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}\right)$$

$$= \pi\left(\frac{2}{3}\cdot 8 - \frac{2}{3}\right)$$

$$= \boxed{\frac{14\pi}{3}}$$

$$\int_*^* \pi\sqrt{u}\,du$$

$$= \pi\,\frac{2}{3}u^{3/2}\Big|_*^* = \pi\cdot\frac{2}{3}(x^2+1)^{3/2}\Big|_0^{\sqrt{3}}$$

$$= \pi\left[\frac{2}{3}(\sqrt{3}^2+1)^{3/2} - \frac{2}{3}(0^2+1)^{3/2}\right]$$

$$= \pi\left[\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}\right]$$

Same

Revolution About Horizontal and Vertical Lines

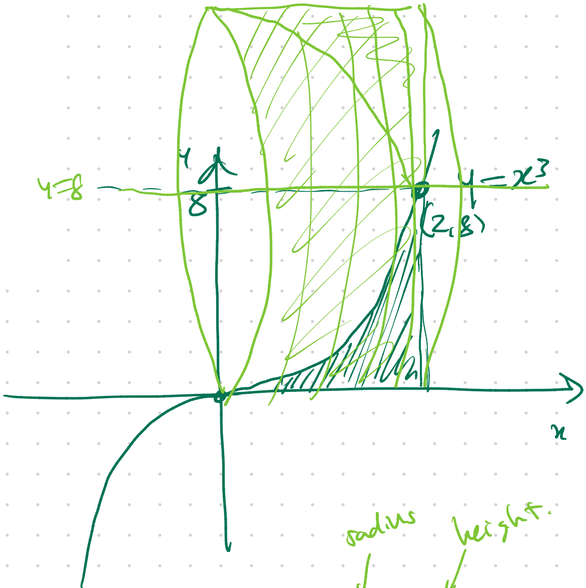
In Exercises 23–26, use the shell method to find the volumes of the solids generated by revolving the regions bounded by the given curves about the given lines.

23. $y = 3x, \quad y = 0, \quad x = 2$

a. The y-axis | b. The line $x = 4$
c. The line $x = -1$ | d. The x-axis
e. The line $y = 7$ | f. The line $y = -2$

24. $y = x^3, \quad y = 8, \quad x = 0$

a. The y-axis | b. The line $x = 3$
c. The line $x = -2$ | d. The x-axis
e. The line $y = 8$ | f. The line $y = -1$



radius height

$$\text{Volume} = \int_0^8 2\pi(8-y)(2-\sqrt[3]{y})\,dy$$

??

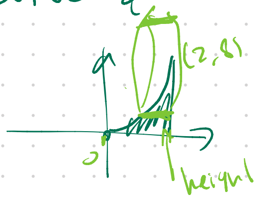
The region bounded by $y = x^3$ and $y = 8$ and $\cancel{x}=0$

Revolve about line $y = 8$.

If Shells then $\int_a^b A(y)\,dy$ & $A(y) = 2\pi r h$

r is the distance between the horizontal line we are rotating around & the particular y-value where the shell is.

height is a horizontal distance between the curve & a vertical line.



$y = x^3$ so $x = \sqrt[3]{y}$

$2 - x = \boxed{2 - \sqrt[3]{y}}$

$r = |8 - y| \quad 0 \le y \le 8$

$r = 8 - y$

$$\frac{d}{dx}\int_1^{x^2} \cos(t)\,dt = \frac{d}{dx}\left(\sin(t)\Big|_1^{x^2}\right)$$

$$= \frac{d}{dx}\left(\sin(x^2) - \sin(1)\right)$$

?

$$\boxed{\int e^{-x^2}\,dx}$$

not ni f.

$$= \cos(x^2) * (x^2)' + 0$$

$$= \cos(x^2) * 2x + 0.$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = \frac{d}{dx}\left[F(t)\Big|_a^{g(x)}\right]$$

$$= \frac{d}{dx}\left(F(g(x)) - F(a)\right)$$

$$= F'(g(x)) * g'(x) + \cancel{0}$$

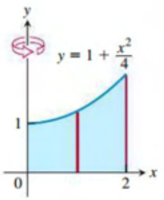
$$= f(g(x)) * g'(x)$$

EXERCISES 6.2

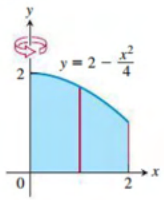
Revolution About the Axes

In Exercises 1–6, use the shell method to find the volumes of the solids generated by revolving the shaded region about the indicated axis.

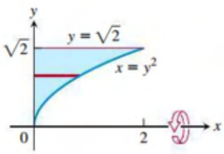
1.



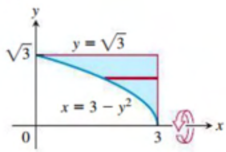
2.



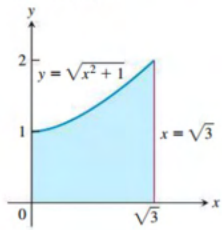
3.



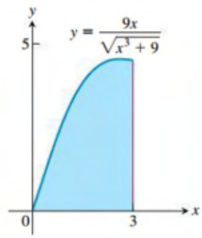
4.



5. The y-axis



6. The y-axis



Revolution About the y-Axis

Use the shell method to find the volumes of the solids generated by revolving the regions bounded by the curves and lines in Exercises 7–12 about the y-axis.

7. $y = x, \quad y = -x/2, \quad x = 2$

8. $y = 2x, \quad y = x/2, \quad x = 1$

9. $y = x^2, \quad y = 2 - x, \quad x = 0, \quad \text{for } x \geq 0$

10. $y = 2 - x^2, \quad y = x^2, \quad x = 0$

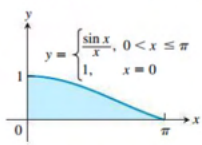
11. $y = 2x - 1, \quad y = \sqrt{x}, \quad x = 0$

12. $y = 3/(2\sqrt{x}), \quad y = 0, \quad x = 1, \quad x = 4$

13. Let $f(x) = \begin{cases} (\sin x)/x, & 0 < x \leq \pi \\ 1, & x = 0 \end{cases}$

a. Show that $x f(x) = \sin x, 0 \leq x \leq \pi$.

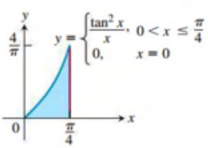
b. Find the volume of the solid generated by revolving the shaded region about the y-axis in the accompanying figure.



14. Let $g(x) = \begin{cases} (\tan x)^2/x, & 0 < x \leq \pi/4 \\ 0, & x = 0 \end{cases}$

a. Show that $x\, g(x) = (\tan x)^2, 0 \leq x \leq \pi/4$.

b. Find the volume of the solid generated by revolving the shaded region about the y-axis in the accompanying figure.



Revolution About the x-Axis

Use the shell method to find the volumes of the solids generated by revolving the regions bounded by the curves and lines in Exercises 15–22 about the x-axis.

15. $x = \sqrt{y}, \quad x = -y, \quad y = 2$

16. $x = y^2, \quad x = -y, \quad y = 2, \quad y \geq 0$

17. $x = 2y - y^2, \quad x = 0$

18. $x = 2y - y^2, \quad x = y$

19. $y = |x|, \quad y = 1$

20. $y = x, \quad y = 2x, \quad y = 2$

21. $y = \sqrt{x}, \quad y = 0, \quad y = x - 2$

22. $y = \sqrt{x}, \quad y = 0, \quad y = 2 - x$

25. $y = x + 2, \quad y = x^2$

a. The line $x = 2$ **b.** The line $x = -1$

c. The x-axis   **d.** The line $y = 4$

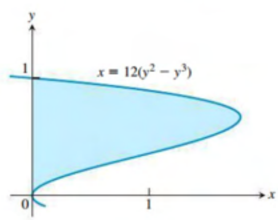
26. $y = x^4, \quad y = 4 - 3x^2$

a. The line $x = 1$ **b.** The x-axis

In Exercises 27 and 28, use the shell method to find the volumes of the solids generated by revolving the shaded regions about the indicated axes.

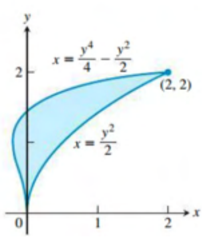
27. a. The x-axis   **b.** The line $y = 1$

c. The line $y = 8/5$ **d.** The line $y = -2/5$



28. a. The x-axis   **b.** The line $y = 2$

c. The line $y = 5$ **d.** The line $y = -5/8$



Revolution About the x-Axis

Use the shell method to find the volumes of the solids generated by revolving the regions bounded by the curves and lines in Exercises 15–22 about the x-axis.

15. $x = \sqrt{y}, \quad x = -y, \quad y = 2$

16. $x = y^2, \quad x = -y, \quad y = 2, \quad y \geq 0$

17. $x = 2y - y^2, \quad x = 0$

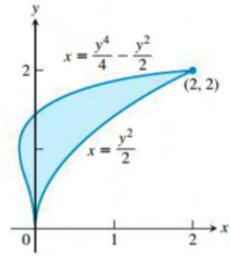
18. $x = 2y - y^2, \quad x = y$

19. $y = |x|, \quad y = 1$

20. $y = x, \quad y = 2x, \quad y = 2$

21. $y = \sqrt{x}, \quad y = 0, \quad y = x - 2$

22. $y = \sqrt{x}, \quad y = 0, \quad y = 2 - x$



Revolution About Horizontal and Vertical Lines

In Exercises 23–26, use the shell method to find the volumes of the solids generated by revolving the regions bounded by the given curves about the given lines.

23. $y = 3x, \quad y = 0, \quad x = 2$

- **a.** The y-axis
- **b.** The line $x = 4$
- **c.** The line $x = -1$
- **d.** The x-axis
- **e.** The line $y = 7$
- **f.** The line $y = -2$

24. $y = x^3, \quad y = 8, \quad x = 0$

- **a.** The y-axis
- **b.** The line $x = 3$
- **c.** The line $x = -2$
- **d.** The x-axis
- **e.** The line $y = 8$
- **f.** The line $y = -1$

Choosing the Washer Method or Shell Method

For some regions, both the washer and shell methods work well for the solid generated by revolving the region about the coordinate axes, but this is not always the case. When a region is revolved about the y-axis, for example, and washers are used, we must integrate with respect to y. It may not be possible, however, to express the integrand in terms of y. In such a case, the shell method allows us to integrate with respect to x instead. Exercises 29 and 30 provide some insight.

29. Compute the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about each coordinate axis using

- **a.** the shell method.
- **b.** the washer method.

30. Compute the volume of the solid generated by revolving the triangular region bounded by the lines $2y = x + 4$, $y = x$, and $x = 0$ about

- **a.** the x-axis using the washer method.
- **b.** the y-axis using the shell method.
- **c.** the line $x = 4$ using the shell method.
- **d.** the line $y = 8$ using the washer method.

In Exercises 31–36, find the volumes of the solids generated by revolving the regions about the given axes. If you think it would be better to use washers in any given instance, feel free to do so.

31. The triangle with vertices (1, 1), (1, 2), and (2, 2) about

- **a.** the x-axis
- **b.** the y-axis
- **c.** the line $x = 10/3$
- **d.** the line $y = 1$

32. The region bounded by $y = \sqrt{x}$, $y = 2$, $x = 0$ about

- **a.** the x-axis
- **b.** the y-axis
- **c.** the line $x = 4$
- **d.** the line $y = 2$

33. The region in the first quadrant bounded by the curve $x = y - y^3$ and the y-axis about

- **a.** the x-axis
- **b.** the line $y = 1$

34. The region in the first quadrant bounded by $x = y - y^3$, $x = 1$, and $y = 1$ about

- **a.** the x-axis
- **b.** the y-axis
- **c.** the line $x = 1$
- **d.** the line $y = 1$

35. The region bounded by $y = \sqrt{x}$ and $y = x^2/8$ about

- **a.** the x-axis
- **b.** the y-axis

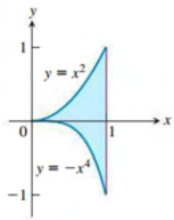
36. The region bounded by $y = 2x - x^2$ and $y = x$ about

- **a.** the y-axis
- **b.** the line $x = 1$

37. The region in the first quadrant that is bounded above by the curve $y = 1/x^{1/4}$, on the left by the line $x = 1/16$, and below by the line $y = 1$ is revolved about the x-axis to generate a solid. Find the volume of the solid by

- **a.** the washer method.
- **b.** the shell method.

40. The region shown here is to be revolved about the y-axis to generate a solid. Which of the methods (disk, washer, shell) could you use to find the volume of the solid? How many integrals would be required in each case? Give reasons for your answers.



41. A bead is formed from a sphere of radius 5 by drilling through a diameter of the sphere with a drill bit of radius 3.

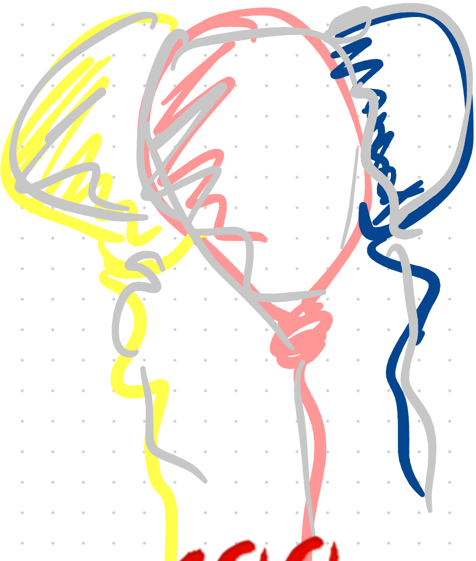
- **a.** Find the volume of the bead.
- **b.** Find the volume of the removed portion of the sphere.

42. A Bundt cake, well known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and the x-axis over the interval $1 \leq x \leq \sqrt{1 + \pi}$. Find the volume of the cake.

43. Derive the formula for the volume of a right circular cone of height h and radius r using an appropriate solid of revolution.

44. Derive the equation for the volume of a sphere of radius r using the shell method.

45. Equivalence of the washer and shell methods for finding volume Let f be differentiable and increasing on the interval $a \leq x \leq b$, with $a > 0$, and suppose that f has a differentiable inverse, f^{-1}. Revolve about the y-axis the region bounded by the graph of f and the lines $x = a$ and $y = f(b)$ to generate a solid. Then the values of the integrals given by the washer and shell methods for the volume have identical values:



Thanks 💯

for a Great

Semester!