Math 1552 Summer 2023	Name (Print):	
Quiz 1	Canvas email:	-NCC
May 25 Time limit: 20 Minutes	Teaching Assistant/Section:	
GT ID:		

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Sign Your Name:

Please clearly organize your work, show all steps, simplify all answers, and BOX your answers.

1. (5 points) Give the **general** anti-derivative of the following function:

$$f(x) = \frac{1}{2} x^{-1/2} - e^{x/5} - \frac{f(x) = \frac{1}{2\sqrt{x}} - e^{x/5} + \frac{1}{\sqrt{9 - x^2}}}{\int 9(1 - [\frac{x}{3}]^2)} = \frac{1}{2} x^{-1/2} - e^{x/5} - \frac{1}{3} \frac{1}{\sqrt{1 - (\frac{x}{3})^2}}$$

$$F(x) = \frac{1}{2} \cdot \frac{\chi^{-1/2 + 1}}{\sqrt{9 - x^2}} - 5 e^{x/5} - \frac{1}{3} \cdot 3 \sin^{-1}(\frac{x}{3}) + C$$

$$= \sqrt{x - 5 e^{x/5} - \sin^{-1}(\frac{x}{3}) + C}$$

2. (5 points) Suppose f(x) is an even function and g(x) is an odd function. If  $\int_{-2}^{2} f(x) dx = 5$  and  $\int_{-2}^{0} g(x) dx = 2$ , find  $\int_{0}^{2} f(x) - g(x) dx$ .

$$\int_{0}^{2} f(x) - g(x) \, dx = \int_{0}^{2} f(x) \, dx - \int_{0}^{2} g(x) \, dx$$

$$= \frac{1}{2} (5) - (-2)$$

$$\int_{-2}^{2} f(x) \, dx = 2 \cdot \int_{0}^{2} f(x) \, dx$$

$$= \frac{5}{2} + 2 = \frac{5}{2} + \frac{4}{2} = \frac{9}{2}$$

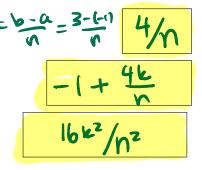
3. (10 points) Suppose  $f(x) = (x + 1)^2$ . Use a general Riemann Sum

$$\lim_{n \to \infty} \sum_{k=1}^n f(x_k^*) \Delta x$$

to evaluate the definite integral of f(x) on the interval [-1,3], by following these steps:

- (a) Find the length of each subinterval  $\Delta x$  in terms of *n*.  $\Delta x = b a = 3 f n$
- (b) Evaluate  $x_k^*$  as the right-hand endpoint of the subinterval.

2= a+ 2 ox = -1+4 B/n



(c) Evaluate the function at  $x_k^*$ , i.e. find  $f(x_k^*)$ . Simplify.

$$f(\chi_{v}) = (-1 + \frac{4}{5} + 1)^{2} = \frac{16v^{2}}{\gamma^{2}}$$

(d) Using the following summation formulas to simplify the sigma notation, find an expression for  $R_n = \sum_{k=1}^n f(x_k^*) \Delta x$  that does not involve sigma's.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^{2} = \frac{n(n+1)(2n+1)}{6}$$

$$R_{n} = \sum_{k=1}^{n} f(x_{k}) \, \delta x = \sum_{k=1}^{n} \frac{16 \, k^{2} \, 4}{n^{2} \, n} = \frac{64}{n^{2}} \sum_{k=1}^{n} k^{2} = \frac{64}{n^{2}} \frac{n(n+1)(2n+1)}{6}$$

$$= \sqrt{\frac{32}{32}} \frac{n(n+1)(2n+1)}{n^{3}}$$

(e) Using the sum you found in the previous step, find the definite integral.

$$\lim_{n \to \infty} R_n = \lim_{n \to \infty} \frac{32}{3} \frac{n(n+i)(2n+i)}{n^2} = \frac{32}{3} \cdot \frac{2}{i} = \frac{64}{3}$$
Chuck w/ FTC  $\int_{-1}^{3} (2i)^2 dx = \int_{0}^{4} u^2 du = \frac{1}{3}u^2 (\frac{4}{5}) = \frac{1}{3} \cdot \frac{64}{3} \cdot \frac{64}{3}$ 
  
 $\frac{1}{5}u - \frac{1}{3}u^2 = \frac{1}{3}u^2$