Math 1552
Summer 2023
Quiz 3 *QUP only*
May 25
Due date: Sunday at 11:59PM

Name (Print): _Key_

Canvas email:

Teaching Assistant/Section:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: _Sal_

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, texbook/course documents, websites, solving tools, or each other (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific.** List the name of anyone who helped you. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (4 points) Fill in the blanks using arbitrary constants $A, B, C, D, \ldots$ (as many as you need) to set up a partial fraction decomposition for the given rational function. Leave any unused boxes blank. *Do not integrate!*

$$\frac{x^2 - 4}{x^2(x^2 + x + 1)^3} = \boxed{\frac{A}{x}} + \boxed{\frac{B}{x^2}} + \boxed{\frac{Cx+D}{x^2+1}} + \boxed{\frac{Ex+F}{(x^2+1)^2}} + \boxed{\frac{Gx+H}{(x^2+1)^3}}$$

2. (8 points) Use partial fractions to find the general anti-derivative of $f(x) = \dfrac{1}{x^2 + x - 2}$.

(Note there is a 3rd page to the QUP version of the quiz this week!)

$$\frac{1}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\Rightarrow A(x-1) + B(x+2) = 1$$

$$\Rightarrow Ax - A + Bx + 2B = 1$$

$$\Rightarrow (A+B)x + (A+2B) = 1$$

So $\quad A+B = 0$

$\begin{cases} A+B=0 \\ -A+2B=1 \end{cases}$

$3B = 1 \Rightarrow B = 1/3$

$A = -1/3$

$$\int \frac{1}{x^2+x-2}\, dx = \int \frac{-1/3}{x+2} + \frac{1/3}{x-1}\, dx$$

$$= \boxed{-\frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C}$$

3. (8 points) Evaluate.

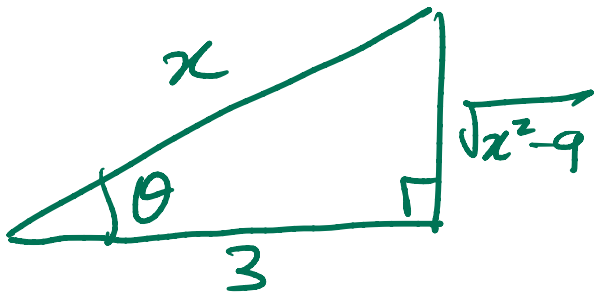$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{\sqrt{(3\sec\theta)^2-9}}{3\sec\theta}\cdot 3\sec\theta\tan\theta\,d\theta$$

$$a^2\sec^2\theta - a^2 = a^2\tan^2\theta$$

trig sub box

$$\boxed{\begin{array}{l} x = 3\sec\theta \\ dx = 3\sec\theta\tan\theta\,d\theta \end{array}}$$

$$= \int \sqrt{9\sec^2\theta-9}\cdot\tan\theta\,d\theta$$

$$= \int \underbrace{\sqrt{9\tan^2\theta}}_{3\tan\theta}\,\tan\theta\,d\theta$$

$$= \int 3\tan^2\theta\,d\theta$$

$$= \int 3(1-\sec^2\theta)\,d\theta$$

$$= \int 3\,d\theta - 3\int \sec^2\theta\,d\theta$$

$$= 3\theta - 3\tan\theta + C$$

$$\frac{x}{3} = \sec\theta = \frac{hyp}{adj}$$

$$\theta = \sec^{-1}(x/3)$$

$$\tan\theta = \frac{opp}{adj} = \frac{\sqrt{x^2-9}}{3}$$

$$= 3\sec^{-1}\left(\frac{x}{3}\right) - \sqrt{x^2-9} + C$$