Math 1552 Summer 2023 **Name (Print):** Key

Quiz 4 *QUP only* **June 15** Teaching Assistant/Section:

Due date: Sunday at 11:59PM

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: ______________

For Question (0.) below please list any **outside resources** you used to help solve quiz problems. You can use calculators, texbook/course documents, websites, solving tools, or each other (e.g., TI-89 calculator, textbook formula sheet on page 281, 3Blue1Brown YouTube video on integrals, WolframAlpha, Symbolab). **Be specific.** List the name of anyone who helped you. If you used no outside resources, write `N/A`.

As always, anything you submit must be your own work. Never submit the work of someone else.

Please clearly organize your work, show all steps, simplify all answers, and $\boxed{\text{BOX}}$ your answers.

0. (1 point) *Full credit for accurately following the directions above.*

N/A

1. (4 points) Find a general formula a_n for the n-th term of the sequence. You do not need to show work on this problem but please put your final answer in the box.

   Hint: be sure to include your starting value for n.

$$0, \frac{3}{5}, \frac{8}{25}, \frac{15}{125}, \frac{24}{625}, \ldots$$

numerators are 1 less than a perfect square "n^2-1"

denominators are powers of 5

$$\boxed{\frac{n^2-1}{5^{n-1}},\ n \geq 1}$$

2. (10 points) Evaluate the improper integral. *Note: there is a third page to the quiz this week.*

$$\int_2^\infty \frac{1}{x\sqrt{x^2-1}}\,dt$$

$$\int_2^N \frac{1}{x\sqrt{x^2-1}}\,dx = \int_*^* \frac{1}{\cancel{\sec\theta}\sqrt{\sec^2\theta-1}}\cdot\cancel{\sec\theta}\tan\theta\,d\theta = \int_*^* \frac{\tan\theta}{\sqrt{\tan^2\theta}}\,d\theta$$

trig sub

$$\boxed{\begin{array}{l} x=\sec\theta \\ dx=\sec\theta\tan\theta\,d\theta \end{array}}$$

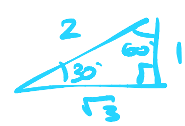
$$= \int_*^* \frac{\cancel{\tan\theta}}{\cancel{\tan\theta}}\,d\theta = \int_*^* 1\,d\theta$$

$$= \theta \Big|_*^*$$

$$= \sec^{-1}(x)\Big|_2^N = \sec^{-1}(N) - \sec^{-1}(2)$$

$\lim\limits_{N\to\infty} \sec^{-1}(N)$ is the value of θ such that $\sec(\theta)$ approaches $+\infty$.

So $\lim\limits_{N\to\infty} \sec^{-1}(N) = \pi/2$



and

$\sec^{-1}(2)$ is the value of θ such that $\sec\theta = 2$, so $\theta = \pi/3$.

$\cos 60° = \frac{1}{2}$

so $\sec 60° = 2$

Finally

$$\lim_{N\to\infty}\int_{\sqrt{2}}^N \frac{1}{x\sqrt{x^2-1}}\,dx = \lim_{N\to\infty} \sec^{-1}(N) - \sec^{-1}(2) = \frac{\pi}{2} - \frac{\pi}{3} = \boxed{\frac{\pi}{6}}$$

3. (6 points) For each sequence, determine the limit of the sequence as n tends to infinity. If the limit diverges, write either DNE, ∞ DNE, or $-\infty$ DNE in the box, as appropriate. You do not have to show your work for problems on this page, but please put your final answer in the box.

(a) $\left\{\left(1+\frac{3}{n}\right)^{2n}\right\}$

So $a_n \to e^6$ $\boxed{e^6}$

$a_n = \left(1+\frac{3}{n}\right)^{2n}$

since $a_n = e^{b_n} = e^{\ln(a_n)}$

$b_n = \ln(a_n) = 2n \ln\left(1+\frac{3}{n}\right) = \dfrac{\ln\left(1+\frac{3}{n}\right)}{1/2n}$

$\lim_{n\to\infty} b_n \underset{\text{L'Hop } \frac{0}{0}}{=} \lim_{n\to\infty} \dfrac{\frac{1}{1+\frac{3}{n}} \cdot \frac{-3}{n^2}}{-1/2n^2} = \lim_{n\to\infty} \frac{1}{1+\frac{3}{n}} \cdot \frac{-3}{\cancel{n^2}} \cdot -2\cancel{n^2} = \frac{1}{1+0} \cdot 3 \cdot 2 = \underline{\underline{6}}$

(b) $\left\{\frac{n!}{e^n}\right\}$

$\boxed{+\infty \text{ DNE}}$

$a_n = \frac{n!}{e^n}$ satisfies

$a_{n+1} = \frac{(n+1)!}{e^{n+1}} = \frac{(n+1)\, n!}{e \cdot e^n} = \frac{n+1}{e} \cdot \frac{n!}{e^n} = \frac{n+1}{e} \cdot a_n$

Notice that $a_{n+1} > 2 \cdot a_n$ whenever $n \geq 4$.

So a_n diverges to $+\infty$ DNE.

(c) $\left\{\frac{(-1)^n}{n+1}\right\}$

$\boxed{0}$

Notice that $a_n = \frac{1}{n+1}$ converges to 0.

So the alternating series $\frac{(-1)^n}{n+1}$ also converges to zero.